

LESSON 15: BODE PLOTS OF TRANSFER FUNCTIONS

ET 438a Automatic Control Systems Technology

Learning Objectives

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After this presentation you will be able to:

- Compute the magnitude of a transfer function for a given radian frequency.
- Compute the phase shift of a transfer function for a given radian frequency.
- Construct a Bode plot that shows both magnitude and phase shift as functions of transfer function input frequency
- Use MatLAB instructions it produce Bode plots of transfer functions.

Construction of Bode Plots

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Bode plots consist of two individual graphs: a) a semilog plot of gain vs frequency b) a semilog plot of phase shift vs frequency. Frequency is the logarithmic axis on both plots.

Bode plots of transfer functions give the frequency response of a control system

To compute the points for a Bode Plot:

- 1) Replace Laplace variable, s , in transfer function with $j\omega$
- 2) Select frequencies of interest in rad/sec ($\omega = 2\pi f$)
- 3) Compute magnitude and phase angle of the resulting complex expression.

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Construction of Bode Plots

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Bode plot calculations- magnitude/phase



Gain: $G(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$

$$\text{dB} = 20 \cdot \log(|G(j\omega)|)$$

Where for a given frequency

$$\mathbf{Y} = y \angle \theta_o$$

$$\mathbf{X} = x \angle \theta_i$$

$$\text{Phase: } \phi = \theta_o - \theta_i$$

To find the magnitude and phase shift of a complex number in rectangular form

given: $\mathbf{z} = a + jb$

$$|\mathbf{z}| = \sqrt{a^2 + b^2}$$

$$\phi = \tan^{-1} \left[\frac{b}{a} \right]$$

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Computing Transfer Function Values

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Example 15-1: A self-regulating tank has a transfer function of the form shown below.

$$\frac{H(s)}{Q(s)} = \frac{G}{1 + \tau \cdot s}$$

The tank has a time constant, $\tau = 1590$ seconds and a gain, $G = 2000 \text{ s/m}^2$. Determine the amplitude and phase shift of the system to a sinusoidal flow input of 0.0001592 Hz

Solution: Substitute values of G , τ , and $j\omega$ into transfer function and compute the gain magnitude and phase shift.

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Example 15-1 Solution (1)

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$$G = 2000 \text{ s/m}^2 \quad \tau = 1590 \text{ s} \quad f = 0.0001592 \text{ Hz}$$

$$\frac{H(j\omega)}{Q(j\omega)} = \frac{2000}{1 + 1590 \cdot j\omega}$$

$$\omega = 2\pi \cdot f \quad \omega = 2\pi \cdot (0.0001592 \text{ Hz}) = 0.001 \text{ rad/s}$$

Place radian frequency into transfer function and compute complex number

$$\frac{H(j0.001)}{Q(j0.001)} = \frac{2000}{1 + 1590 \cdot j0.001} = \frac{2000}{1 + j1.590} = 566.9 - j901$$

Convert the result to polar form to find magnitude and phase shift

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Example 15-1 Solution (2)

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Complex gain $G(j0.001) = \frac{2000}{1+1590 \cdot j0.001} = \frac{2000}{1+j1.590} = 566.9 - j901$

$$|G(j\omega)| = \sqrt{a^2 + b^2} \quad \phi = \tan^{-1} \left[\frac{b}{a} \right]$$

Magnitude Calculation

$$a = 566.9 \quad b = -901$$

$$|G(j0.001)| = \sqrt{566.9^2 + (-901)^2}$$

$$|G(j0.001)| = 1065$$

$$\text{dB} = 20 \cdot \log(1065) = 60.5 \text{ dB} \quad \leftarrow \text{Ans}$$

Phase shift

$$\phi = \tan^{-1} \left[\frac{b}{a} \right] = \tan^{-1} \left[\frac{-901}{566.9} \right] = -58^\circ \quad \leftarrow \text{Ans}$$

At 0.001 rad/sec, the system has a gain of 60.5 dB and the output changes in height lag the flow changes by 58 degrees

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Constructing Bode Plots Using MatLAB

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MatLAB has control system toolbox functions for defining Linear Time-invariant systems (LTI) and constructing the Bode plots.

Use tf and bode functions to create LTI and plot. Introducing zpk function

`sys = zpk(z,p,k)` Turns arrays of zeros, poles and gains into LTI called sys

Where `z` = array of transfer function zeros

`p` = array of transfer function poles

`k` = array of transfer function gains

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Constructing Bode Plots Using MatLAB

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Example 15-2: Construct the Bode plot for the given transfer function shown in factored form using MatLAB control toolbox functions.

$$\frac{V_o(s)}{V_i(s)} = \frac{0.005 \cdot s}{(0.001 \cdot s + 1)(0.001 \cdot s + 1)}$$

Solution: Transfer function has one zero at $s=0$ and two poles at $s=-1/0.001=-1000$

Dividing the transfer function denominator and numerator by 0.001 places it into standard form

$$\frac{V_o(s)}{V_i(s)} = \frac{0.005 \cdot s \cdot \left[\frac{1}{0.001} \right]}{(0.001 \cdot s + 1)(0.001 \cdot s + 1) \cdot \left[\frac{1}{0.001} \right]} = \frac{s \cdot \left[\frac{0.005}{0.001} \right]}{\left(\frac{0.001}{0.001} \cdot s + \frac{1}{0.001} \right) \left(\frac{0.001}{0.001} \cdot s + \frac{1}{0.001} \right)} = \frac{5 \cdot s}{(s + 1000)(s + 1000)}$$

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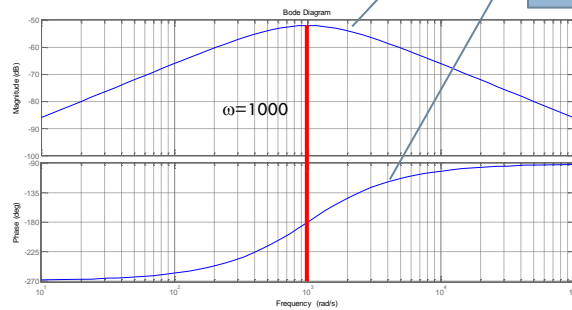
Constructing Bode Plots Using MatLAB

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MatLAB code to produce Bode plot of given transfer function

Enter at command prompt or into m-file

```
k= [5]
p=[1000 1000]
z=[0]
sys=zpk(z,p,k)
bode(sys)
```



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Constructing Bode Plots Using MatLAB

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The `bode(sys)` function can plot more than one transfer function on the same figure axis. The figure produced by the `bode(sys)` function can be copied and pasted into wordprocessors and other programs. To plot more than one transfer function use the following syntax: `bode(sys1,sys2,...)`.

Example 15-3: Compare the Bode plots of the transfer function given in Example 15-2 to the function given below. Use MatLAB to generate the Bode plots on a single set of axis. Note: the only difference is the gain is increased by a factor of 100.

$$\frac{V_o(s)}{V_i(s)} = \frac{500 \cdot s}{(s+1000)(s+1000)}$$

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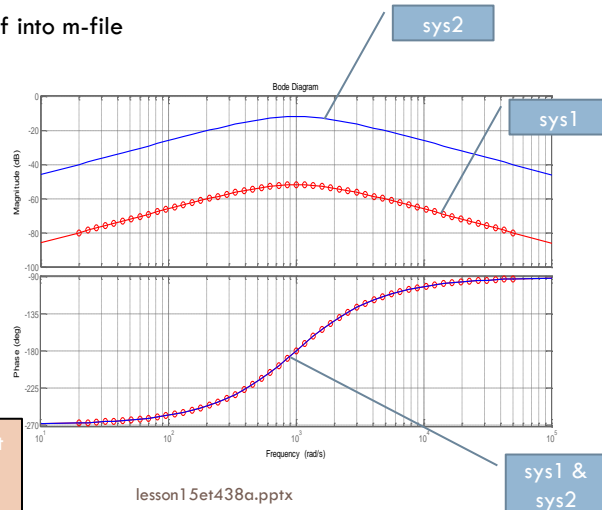
Constructing Bode Plots Using MatLAB

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Enter at command prompt or into m-file

```
k= [5]
p=[1000 1000]
z=[0]
sys1=zpk(z,p,k)
k1=[500]
sys2=zpk(z,p,k1)
bode(sys1,'ro-
',sys2,'b-')
```

This changes plot styles
(blue solid line)



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End Lesson 15: Bode Plots of Transfer Functions

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