Lesson 12a: Three Phase Induction Motors

ET 332b

Ac Motors, Generators and Power Systems

Learning Objectives

After this presentation you will be able to:

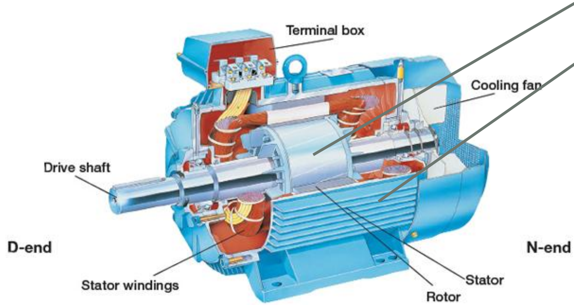
- Explain how a three-phase induction motor operates
- Compute the synchronous speed of an induction motor and the slip between motor rotor and stator magnetic field
- Compute the power that crosses that air gap of an induction motor
- Explain how the parameters of an induction motor circuit model relate to its performance
- Identify model equations

Three-Phase Induction Motors

Motor Construction

Rotor

Stator

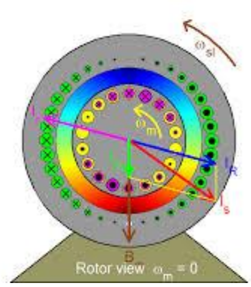


Stator - magnetic structure (iron core) and winding that create magnetic field. Connected to three-phase voltages

Rotor - iron core and conductors that rotate and drive the shaft of the motor. Conductors can be either copper bars (squirrel cage) or wound coils (wound-rotor)

Three-Phase Induction Motors

The three-phase voltages V_a, V_b and V_c create fluxes that add in space and time to create a rotating magnetic field without physical motion.



Flux wave rotates at a speed given by:

$$n_s = \frac{120 \cdot f}{P}$$

Where n_s = synchronous speed
f = ac voltage frequency
P = number of poles
(not pole pairs)

Synchronous Speed

Example 12a-1: Four pole motor operating on a 60 Hz system. What is the speed at which the magnetic field rotates

$$n_s = \frac{120 \cdot f}{P} \qquad \begin{aligned} P &= 4\,\text{poles} \\ f &= 60\,\text{Hz} \end{aligned} \qquad n_s = \frac{120 \cdot 60}{4} = 1800\,\text{rpm}$$

When supplied from 60 Hz system, n_s is multiple of 60

Induction Motor Operation

For an Induction Motor to Rotate

3-phase voltages produce rotating magnetic field in stator

↓

Current is induced in rotor by moving magnetic field

↓

Induced current in rotor produces a magnetic field in rotor

↓

Field in rotor interacts with the field in the stator to produce torque (rotor "chases" stator field)

Slip and Slip Speed

To induce current in rotor there must be a speed difference between the rotor and the rotating magnetic field. This speed difference is called slip speed

$$n_{sl} = n_s - n_r$$

Where n_{sl} = slip speed
n_s = synchronous speed
n_r = rotor speed

Define slip as per unit value

$$s = \frac{n_s - n_r}{n_s}$$

Slip increases as load increases

Slip and Developed Torque

At start up $n_r = 0$. Assuming n_s = 1800 RPM determine the slip

$$s = \frac{n_s - n_r}{n_s} = s = \frac{1800 - 0}{1800} = 1$$

Slip is 1 at locked rotor (startup)

At full load torque motor spins at rated speed

Rated speed n_r =1750 rpm: typical for 4 pole induction motor

$$s = \frac{n_s - n_r}{n_s} = s = \frac{1800 - 1750}{1800} = 0.028$$

Rated slips vary 0.02-0.05 of n_s.

Slip at No-load

Rotor spins at nearly n_s, so n_r=1798 typical for unloaded 4 pole motor

$$s = \frac{n_s - n_r}{n_s} = s = \frac{1800 - 1798}{1800} = 0.001$$

Slip is near zero when there is no-load on the motor

Three-Phase Induction Motors

Advantages

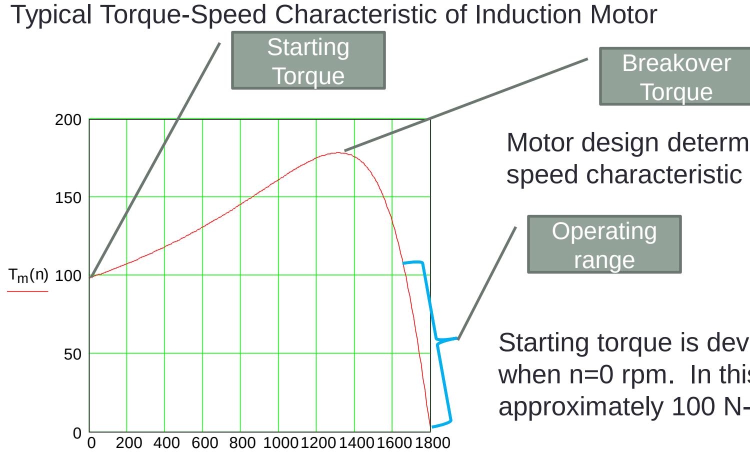
- Smooth Power Transfer
  - Power almost constant in 3-phase systems
  - Power pulsates in single phase motors
- Simple Construction
  - No brushes or other high maintenance parts

Disadvantages

- Can not easily control speed

Induction Motor Torque-Speed Characteristic

Typical Torque-Speed Characteristic of Induction Motor



Motor design determines speed characteristic shape

Starting torque is developed when n=0 rpm. In this case approximately 100 N-m

Slip Speed & Rotor Voltage/Frequency

Difference between speed of rotating magnetic field and rotor called slip speed

$$n_{sl} = n_s - n_r$$

Where n_{sl} = slip speed

Slip speed increases as load increases and rotor frequency is a function of slip

$$f_r = \frac{s \cdot P \cdot n_s}{120}$$

Where: n_s = synchronous speed
s = p.u slip
f_r = frequency of rotor induced voltage

Slip Speed & Rotor Voltage/Frequency

With the rotor blocked n=0, s=1

$$f_r = \frac{P \cdot n_s}{120} = f_{stator} = f_{BR}$$

Where: f_{stator} = stator voltage frequency
f_{BR}= blocked rotor frequency

At startup stator voltage frequency and rotor voltage frequency are equal

In operation slip not equal 1, so generally..

$$f_r = s \cdot f_{BR}$$
$$E_r = s \cdot E_{BR}$$

Induced V max at s=1

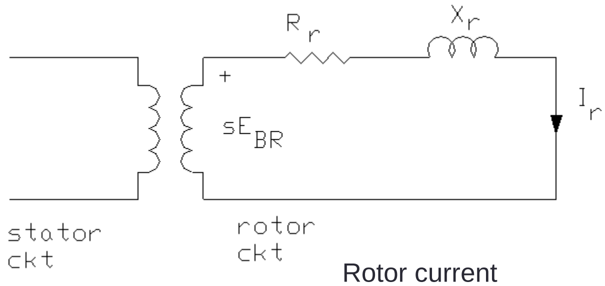
Where: E_r = voltage induced in rotor at slip s
E_{BR} = voltage induced with n=0 (Blocked rotor)

Motor Rotor Circuit Model

Motor has resistance and inductive reactance. X_L depends on f so

$x_r = 2\pi \cdot f_r \cdot L_r$ $\quad x_r = 2\pi \cdot s \cdot f_{BR} \cdot L_r$ $\quad x_r = s \cdot X_{BR}$

Rotor reactance in terms of blocked rotor inductance



Rotor current

Rotor Impedance

$z_r = R_r + j \cdot x_r = R_r + j \cdot s \cdot X_{BR}$

$$I_r = \frac{s \cdot E_{BR}}{R_r + j \cdot x_r} = \frac{s \cdot E_{BR}}{R_r + j \cdot s \cdot X_{BR}}$$

Motor Rotor Circuit Model

Some algebra gives

$$I_r = \frac{E_{BR}}{\left(\frac{R_r}{s}\right) + j \cdot X_{BR}}$$

Rotor current depends on slip which is related to motor speed

Phase angle of Z_r depends on slip (R changes), so impedance angle and F_p changes with motor slip. This means rotor current magnitude and phase angle change with slip

Rotor current magnitude

$$|I_r| = \frac{E_{BR}}{\sqrt{\left(\frac{R_r}{s}\right)^2 + X_{BR}^{\ 2}}}$$

Rotor current phase angle

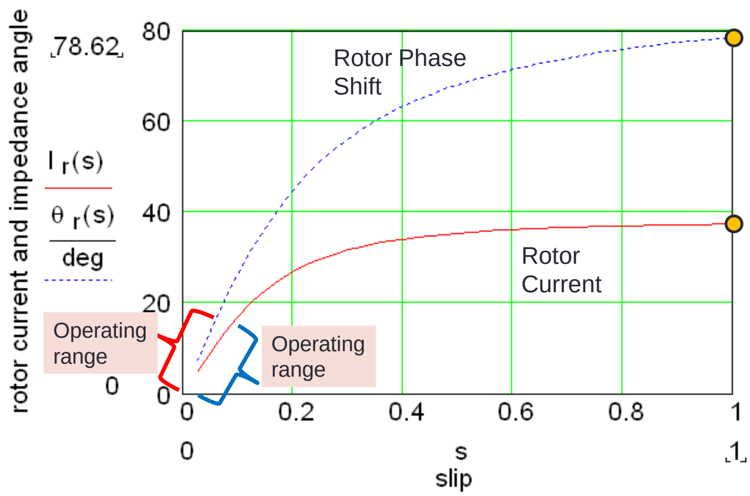
$$\theta_r = \tan^{-1}\left(\frac{X_{BR}}{\left(\frac{R_r}{s}\right)}\right)$$

Rotor power factor

$$F_{pr} = \cos(\theta_r)$$

Where θ_r = rotor current angle

Motor Rotor Circuit Model



Induction Motor Air Gap Power

Define power transferred across the air gap in the induction motor

$$\bar{S}_{gap} = \bar{E}_{BR} \cdot \bar{I}_r^{\,*}$$

Where $\bar{E}_{BR} = E_{BR}\angle 0^\circ \qquad \bar{I}_r = I_r\angle -\theta_r^\circ$

In rectangular form $\bar{S}_{gap} = |\bar{E}_{BR}| \cdot |\bar{I}_r| \cdot \cos(\theta_r) + j \cdot |\bar{E}_{BR}| \cdot |\bar{I}_r| \cdot \sin(\theta_r)$

With the following components

$$P_{gap} = |\bar{E}_{BR}| \cdot |\bar{I}_r| \cdot \cos(\theta_r)$$

$$Q_{gap} = |\bar{E}_{BR}| \cdot |\bar{I}_r| \cdot \sin(\theta_r)$$

P_{gap} = active power providing shaft power, friction, windage, and rotor resistance losses.

Q_{gap} = reactive power that oscillates across air gap

Rotor F_p and the magnitude of the I_r determine gap active power, P_{gap}
E_{BR} is assumed to be constant because it is proportional to the flux density which is assumed to be constant

Active Power Across Air-Gap

Components

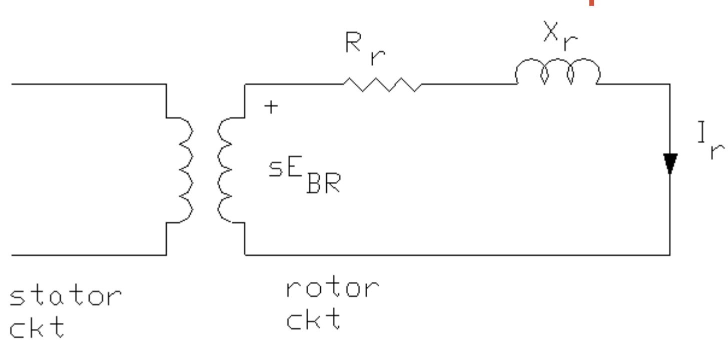
$$P_{gap} = P_{mech} + P_{rcl}$$

Where

P_{mech} = active power converted to shaft power
P_{rcl} = rotor conductor losses

Total 3-phase rotor losses $$P_{rcl} = 3 \cdot I_r^{\ 2} \cdot R_r$$

Active Power Across Air-Gap



Total gap power $$P_{gap} = \frac{3 \cdot I_r^{\ 2} \cdot R_r}{s}$$

Active Power Across Air-Gap

Slip related to the amount of mechanical load on motor. More mechanical load more active power across gap

Combine power balance equations with definitions of P_{gap} and P_{rcl}

$$P_{mech} = \frac{3 \cdot I_r^{\,2} \cdot R_r \cdot (1-s)}{s}$$

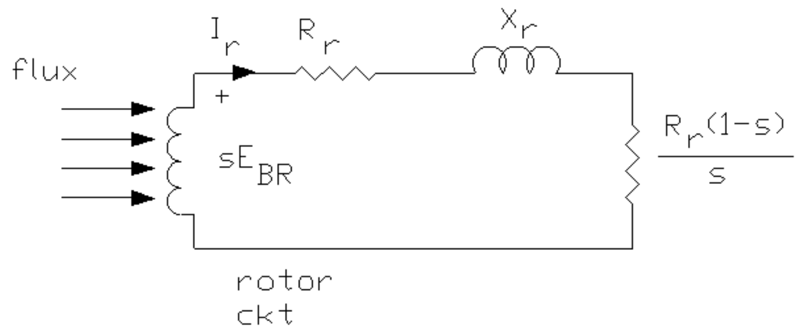
Active Power Across Air-Gap

Rotor resistance effects the amount of mechanical power developed

Divide $\frac{R_r}{s}$ Into two parts: rotor loss resistance and the resistance that represents mechanical load

$$\frac{R_r}{s} = \frac{R_r \cdot (1-s)}{s} + R_r$$

Active Power Across Air-Gap



Per phase model of the rotor

Developed Torque and Mechanical Power

Mechanical power in terms of n_s and n_r

$$s = \frac{n_s - n_r}{n_s} \qquad s = 1 - \frac{n_r}{n_s} \qquad \text{so } 1 - s = \frac{n_r}{n_s}$$

Substitute into the previous equation for mechanical power

$$P_{mech} = \frac{3 \cdot I_r^2 \cdot R_r \cdot n_r}{s \cdot n_s}$$

Developed Torque and Mechanical Power

Mechanical power related to rotor resistance and current
To find torque divide mechanical power by speed

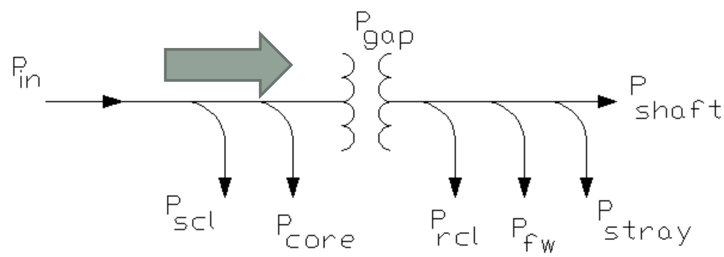
$$T_d = \left(\frac{180 \cdot R_r}{2\pi \cdot s \cdot n_s}\right) \cdot \left[\frac{{E_{BR}}^2}{\left(\frac{R_r}{s}\right)^2 + {X_{BR}}^2}\right] \quad \text{N-m}$$

Developed Torque and Mechanical Power

This equation assumes an ideal stator – no losses.

Used to generate torque-speed curves – rotor resistance effects the developed power and, therefore torque

Motor Losses Efficiency & Power Factor



P_{in} = Electric power in to motor
P_{scl} = Stator conductor losses; P_{core} = Core losses;
P_{rcl} = Rotor conductor losses; P_{fw} = Friction and windage;
P_{stray} = Stray losses;
P_{shaft} = Mechanical power output (rated HP);
P_{mech}= Electric power converted to mechanical power in rotor.

Developed Torque and Mechanical Power

Power converted

$$P_{gap} = \frac{P_{rcl}}{s}$$

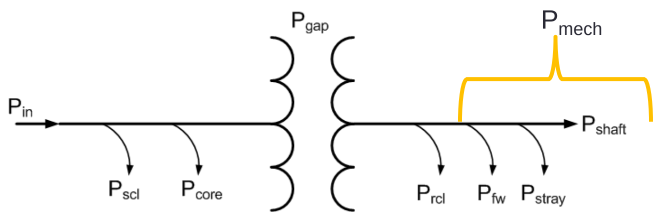
Total active power across air gap
P_{rcl} = rotor conductor losses

$$P_{mech} = P_{gap} \cdot (1 - s)$$

Portion of active gap power converted to mechanical power
$P_{mech} = P_{shaft} + P_{fw} + P_{stray}$

Power Balance Equations

Power in must equal power out plus losses



Rated shaft power (HP): $P_{shaft} = P_{mech} - P_{fw} - P_{stray}$

From stator side: $P_{gap} = P_{in} - P_{scl} - P_{core}$

Power Balance Equations

Total apparent electric power in $S_{in} = \sqrt{3} \cdot I_L \cdot V_{LL}$

Find P_{in} from F_p and S_{in} values $F_p = \frac{P_{in}}{S_{in}}$

Also, given a motor efficiency at an output level

$$\eta = \frac{P_o}{P_{in}} = \frac{P_{shaft}}{P_{in}}$$

Can find P_{in}

Example 12a-2

A 3-phase 60 Hz, 75 Hp, 4 pole motor operates at a rated terminal voltage of 230 V Under rated conditions it draws a line current of 186 A and has an efficiency of 90%. The following losses are measured:

Core losses = 1273 W Stator conductor losses = 2102 W

Rotor conductor losses = 1162 W

Find: a) the input power
- b) the total losses
- c) the air gap power
- d) the shaft speed
- e) the motor power factor
- f) combined mechanical losses

Example 12a-2 Solution (1)

a) Find input power

$\eta = \frac{P_{shaft}}{P_{in}} \Rightarrow \frac{P_{shaft}}{\eta} = P_{in}$

$P_{shaft} = 75\ Hp \quad \eta = 90\%$

$1\ Hp = 746 W$

$P_{shaft} = (75\ Hp)(746\ W/Hp)$

$P_{shaft} = 55{,}950\ W$

$P_{in} = \frac{55{,}950}{\frac{90\%}{100}} = 62{,}167 W$ Ans

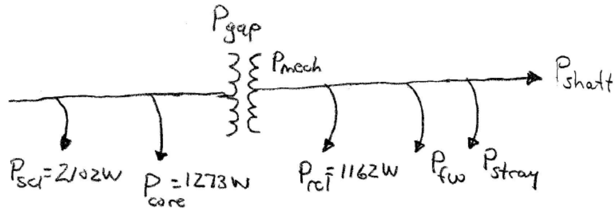
b) Find the total losses

Losses are the difference between the input and output powers

$P_{in} - P_{shaft} = P_{Loss} \quad 62{,}167 W - 55{,}950 W = 6217 W$ Ans

Example 12a-2 Solution (2)

c) Find the gap power



$$P_{gap} = P_{in} - P_{scl} - P_{core}$$

$$P_{gap} = 62{,}167W - 2102W - 1273W$$

$$P_{gap} = 58{,}792\ W$$

Ans

Example 12a-2 Solution (3)

d) Find shaft speed

$$P_{gap} = \frac{P_{rcl}}{s} \qquad P_{rcl} = \text{rotor conductor power loss}$$

From above $\quad s = \dfrac{P_{rcl}}{P_{gap}} = \dfrac{1162W}{58{,}792W} \qquad s = 0.01976$

Find synchronous speed $\quad f = 60\ Hz \qquad P = 4$

$$n_s = \frac{120f}{P} = \frac{120(60)}{4} \qquad \frac{n_s - n_r}{n_s} = s \Rightarrow 1 - \frac{n_r}{n_s} = s$$

$$n_s = 1800\ RPM$$

Example 12a-2 Solution (4)

$(1-s)n_s = n_r$ $\quad n_r = (1-0.01976)(1800)$

$n_r = 1764$ RPM ← Ans

e) Motor power factor – ratio of apparent power to active power

$S_{in} = \sqrt{3}\, V_{LL} I_L$ $\quad I_L = 186$ A $\quad V_{LL} = 230$ V $\quad S_{in} = \sqrt{3}(230\text{V})(186\text{A})$

$S_{in} = 74097$ VA

$P_{in} = 62167$ W

$$F_p = \frac{P_{in}}{S_{in}} = \frac{62{,}167 \text{ W}}{74{,}097 \text{ VA}}$$

$F_p = 0.84$ Lag ← Ans

Example 12a-2 Solution (5)

f) Combined mechanical losses

$P_{mech} - P_{shaft} = P_{fw} + P_{stray}$ TOTAL MECHANICAL LOSSES

$P_{mech} = P_{gap} - P_{rcl} = 58{,}792 \text{ W} - 1162 \text{ W} = 57{,}630 \text{ W}$

$P_{shaft} = 55{,}950$ W

$P_{mech} - P_{shaft} = 57{,}630 - 55{,}950 \text{ W} = 1680$ W ← Ans

Example 12a-3

A 3-phase 230V, 25 HP, 60Hz, 4 pole motor rotor absorbs 20,200 W when supplying an unknown shaft load. The rotor copper losses are measured at 975 W when supplying this load. The friction and windage losses are known to be 250 W. Determine

a) the shaft speed;

b) mechanical power developed;

c) torque developed in the rotor;

d) shaft torque;

e) percent of rated horsepower that the motor is delivering.

Example 12a-3 Solution (1)

a) Motor speed

$$P_{gap} = \frac{P_{rcl}}{s} \Rightarrow s = \frac{P_{rcl}}{P_{gap}}$$

$$s = \frac{975\text{W}}{20{,}200\text{W}} = 0.0483 \qquad n_s = \frac{120f}{P} = \frac{120(60\text{Hz})}{4} = 1800\text{ Rpm}$$

$$n_r = (1-s)n_s \qquad n_r = (1-0.0483)(1800\text{ Rpm}) = 1713\text{ RPM}$$

Ans

b) Mechanical power developed is P_{gap} less rotor conductor losses

$$P_{mech} = P_{gap} - P_{rcl}$$

$$P_{mech} = 20{,}200\text{W} - 975\text{W} = 19{,}225\text{W}$$

Ans

Example 12a-3 Solution (2)

c) Compute developed torque in lb-ft

$$P_{mech} = T_D \omega \quad N\text{-}m$$

$$P_{mech} = \frac{T_D n_r}{7.04} \quad lb\text{-}ft$$

$$T_D = \frac{7.04 P_{mech}}{n_r} = \frac{7.04(19{,}225\ W)}{1713\ RPM} \qquad T_D = 79\ lb\text{-}ft$$

Ans

d) Compute shaft torque with shaft power

$$P_{shaft} = P_{mech} - P_{fw} - P_{stray} \qquad P_{stray} = 0\ W$$

$$P_{shaft} = 19{,}225\ W - 250\ W = 18{,}975\ W$$

$$T_{shaft} = \frac{7.04\ P_{shaft}}{n_r}$$

$$T_{shaft} = \frac{7.04(18{,}975\ W)}{1713\ rpm}$$

$$T_{shaft} = 77.98\ lb\text{-}ft$$

Ans

Example 12a-3 Solution (3)

e) Percent Load

$$P_{shaft(rated)} = (746\ W/HP)(25\ HP) = 18{,}650\ W$$

$$\%\ Rated = \frac{P_{shaft} \times 100\%}{P_{shaft(rated)}} = \frac{18{,}975\ W}{18{,}650\ W} \times 100\%$$

$$\%\ Rated = 101.7\% \quad \text{OVERLOADED}$$

Full Induction Motor Model

Per phase circuit similar to transformer

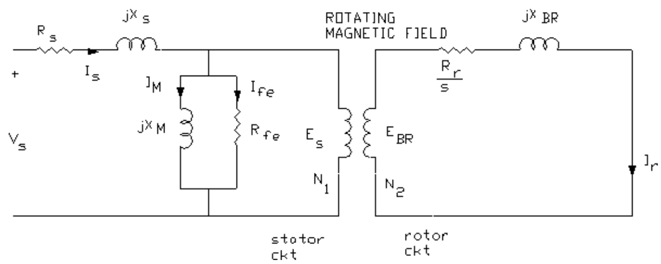
V_s = stator voltage (line voltage) R_{fe} = equivalent core resistance

R_s = stator winding resistance Rr = actual rotor resistance

X_s = stator leakage reactance X_{BR} = actual blocked-rotor reactance

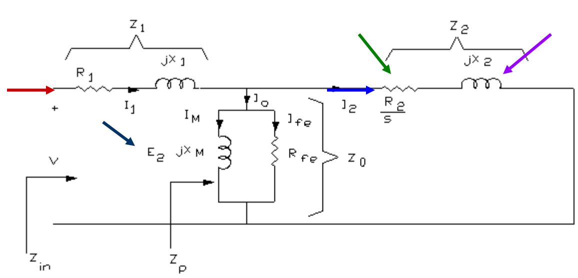
X_M = stator core magnetizing reactance

a = N1/N2 ratio of stator to rotor turns



Full Induction Motor Model

Per phase motor model-rotor quantities referred to stator.



Where: I_1 = stator current $I_2 = I_r/a^2$: rotor current referred to stator

$R_2 = R_r a^2$ rotor resistance referred to stator

$X_2 = X_{BR} a^2$ blocked rotor reactance referred to the stator

$E_2 = E_s a$ blocked rotor voltage referred to the stator

Full Induction Motor Model

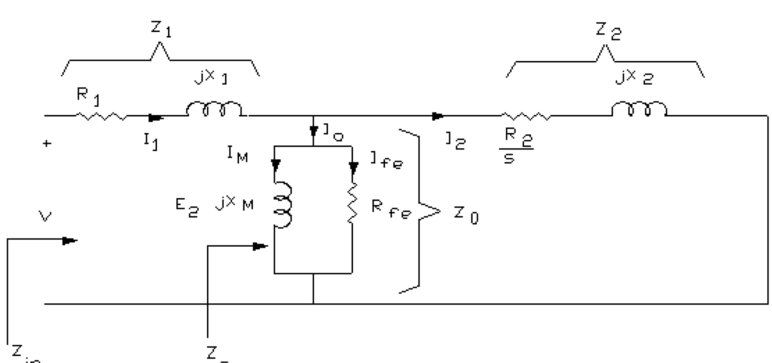
Remember, R_2 can be written as:

$$R_2 = R_r \cdot a^2 = a^2\left[\frac{R_r \cdot (1-s)}{s}\right] + a^2 \cdot R_r$$

The power, torque speed and efficiency can now be found analytically from the model if input, output and model parameters are known.

Full Induction Motor Model

Use circuit analysis techniques to determine motor performance



Full Induction Motor Model

$$\overline{Z}_2 = \frac{R_2}{s} + j \cdot X_2$$

Rotor impedance

$$\overline{Z}_0 = \frac{R_{fe} \cdot j \cdot X_M}{R_{fe} + j \cdot X_M}$$

Parallel combination of core values

$$\overline{Z}_P = \frac{\overline{Z}_2 \cdot \overline{Z}_0}{\overline{Z}_2 + \overline{Z}_0}$$

$$\overline{Z}_{in} = \overline{Z}_P + \overline{Z}_1 \quad \text{where} \quad Z_1 = R_1 + j \cdot X_1$$

Total motor model impedance (per phase)

$$\overline{I}_1 = \frac{\overline{V}}{\overline{Z}_{in}}$$

Stator current

$$\overline{E}_2 = \overline{I}_1 \cdot \overline{Z}_P$$

Induced rotor voltage referred to stator

Full Induction Motor Model

$$\overline{I}_2 = \frac{\overline{E}_2}{\overline{Z}_2}$$

Rotor current referred to stator

Total power relationships

$$P_{scl} = 3 \cdot \left|\overline{I}_1\right|^2 \cdot R_1$$

Total stator conductor losses

$$P_{rcl} = 3 \cdot \left|\overline{I}_2\right|^2 \cdot R_2$$

Total rotor conductor losses

$$P_{gap} = \frac{P_{rcl}}{s}$$

$$P_{mech} = P_{rcl} \cdot \left[\frac{(1-s)}{s}\right]$$

Note: all power equations are for total three-phase power

Full Induction Motor Model

$$P_{shaft} = P_{mech} - P_{fw} - P_{stray}$$

Shaft power is mechanical power developed less mechanical losses

$$T_D = \frac{7.04 \cdot P_{mech}}{n_r} \quad (\text{lb-ft})$$

Rotor developed torque. Where n_r = rotor speed

$$T_{shaft} = \frac{7.04 \cdot P_{shaft}}{n_r} \quad (\text{lb-ft})$$

Shaft torque

Finally

$$P_{core} = \frac{3 \cdot |\overline{E}_2|^2}{R_{fe}}$$

The stator core losses are dependent on the voltage

End Lesson 12a: Three Phase Induction Motors

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