Lesson 6: Sampling Analog Signals

ET 438b Sequential Control and Data Acquisition

Department of Technology

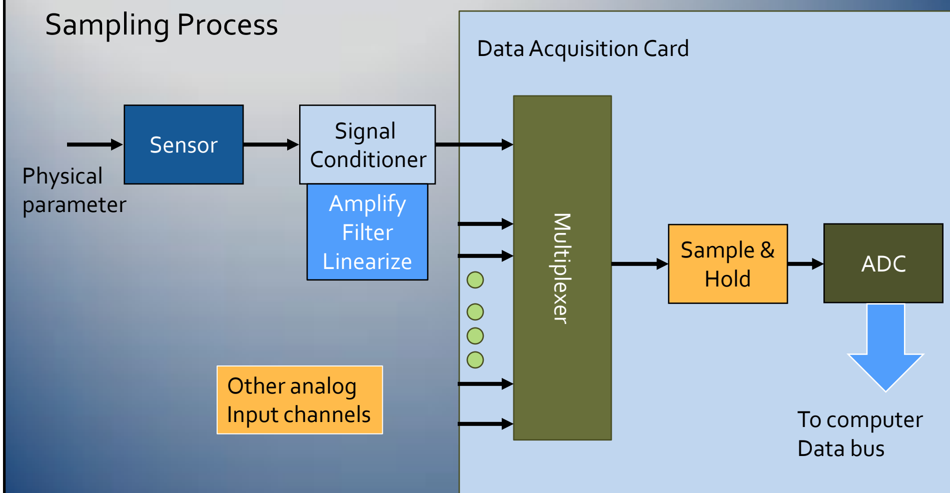
Learning Objectives

After this presentation you will be able to:

- Identify the steps in sampling an analog signal
- Indentify the frequency spectrum of a sampled signal
- Determine the minimum sampling rate of an analog signal
- Determine if a sampled signal contains aliased signals.

Sampled Signals

Sampling Process



Sampled Signals-Representation of Signal

Analog Signal - defined at every point of independent variable For most physical signals independent variable is time

Sampled Signal - Exists at point of measurement. Sampled at equally spaced time points, T_s called sampling time. ($1/T_s = f_s$), sampling frequency

Analog Example

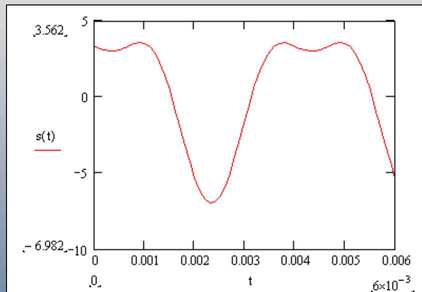
$$s(t) = 5 \cdot \sin(2\pi \cdot 250 \cdot t + 60^{\circ}) + 2 \cdot \cos(2\pi \cdot 500 \cdot t + 120^{\circ})$$

Sampled Example

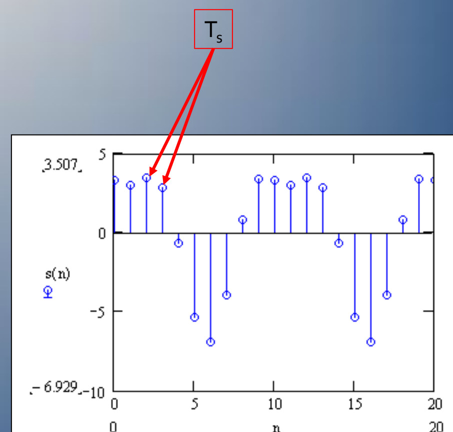
$$s(n) = 5 \cdot \sin(2\pi \cdot 250 \cdot T_s \cdot n + 60^{\circ}) + 2 \cdot \cos(2\pi \cdot 500 \cdot T_s \cdot n + 120^{\circ})$$

n=sample number

Sampled Data Examples



Representation of analog signal

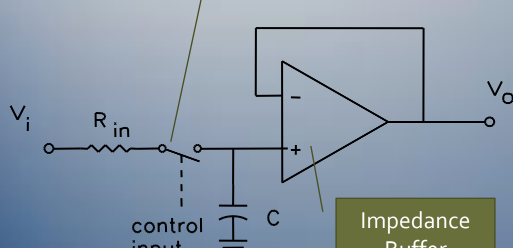


Representation of sampled analog signal

Sample and Hold Operation

Sample and Hold Circuit

Control input operates solid –state switch at sampling rate f_s



Impedance Buffer

Operating Modes
tracking = switch closed
hold= switch open

Sample and Hold Parameters

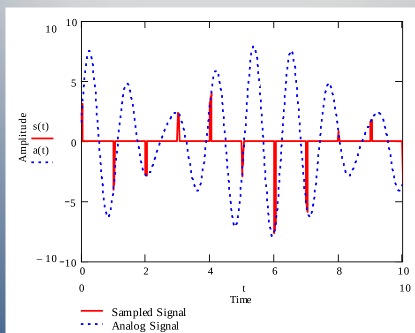
Acquisition Time - time from instant switch closes until V_i within defined % of input. Determined by input time constant $\tau = R_{in}C$ 5τ value = 99.3% of final value

decay rate - rate of discharge of C when circuit is in hold mode

aperture time - time it takes switch to open.

Sample and Hold Signals

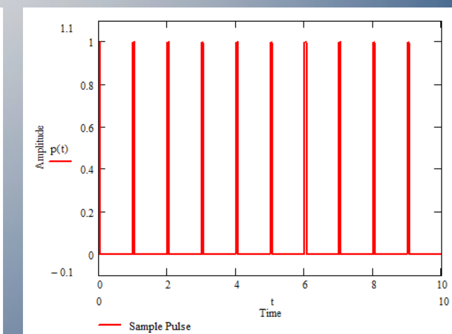
Pulse generator closes switch and captures signal value



Analog and sampled signal

Amplitude Modulated

s(t)=p(t)·a(t)



Pulse generator output

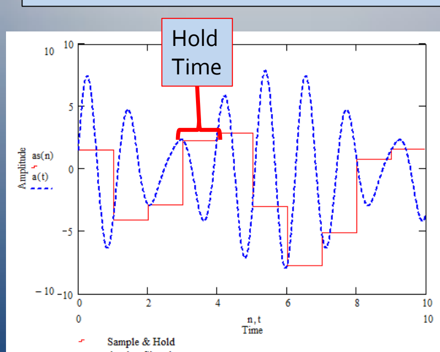
Sample and Hold Output

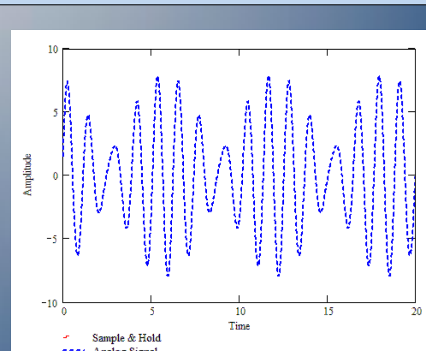
Sample must be held while digital conversion takes place. Total time to digitize

$t_c = t_a + t_d$ Where t_c = total conversion time

t_a = total acquisition time

t_d = total digital conversion time





Frequency Spectrum

Sampling is modulation. Shifts all signal frequency components and generates harmonics

f_c=1000 Hz f_{l1}=50 Hz f_{l2}=25 Hz

$$v(t) = 1\cdot\sin(2\pi\cdot 1000\cdot t)\left[1\cdot\sin(2\pi\cdot 50\cdot t) + 1\cdot\sin(2\pi\cdot 25\cdot t)\right]$$

Carrier: $1\cdot\sin(2\pi\cdot 1000\cdot t)$

Information: $\left[1\cdot\sin(2\pi\cdot 50\cdot t) + 1\cdot\sin(2\pi\cdot 25\cdot t)\right]$

Modulation produces sums and differences of carrier and information frequencies

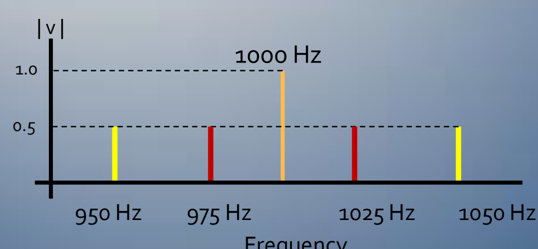
$f_{h1} = f_c \pm f_{l1}$ for the 1st information frequency
$f_{h2} = f_c \pm f_{l1}$ for the 2nd information frequency
$f_{hi} = f_c \pm f_{li}$ for the i-th information frequency

Frequency Spectrum

Frequency Components

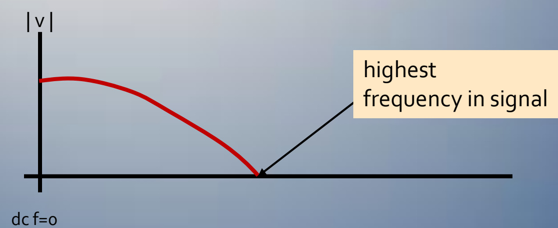
$f_{h1} = f_c \pm f_{l1}$ =1000 Hz ± 50 Hz = 1050 Hz and 950 Hz
$f_{h2} = f_c \pm f_{l1}$ =1000 Hz ± 25 Hz = 1025 Hz and 975 Hz

Frequency Spectrum Plot



Frequency Spectrum

Complex signals usually have a frequency spectrum that is wider. Can be visualized with continuous f plot and found with an Fast Fourier Transform (FFT)



Frequency spectrum of input signals sample & hold must be known to accurately reproduce original signal from samples

Nyquist Frequency and Minimum Sampling Rate

To accurately reproduce the analog input data with samples the sampling rate, f_s, must be twice as high as the highest frequency expected in the input signal. (Two samples per period) This is known as the Nyquist frequency.

$$f_{s(min)} = 2f_h$$

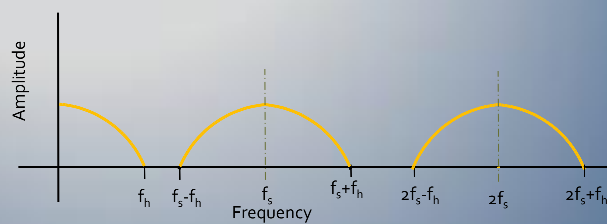
Where f_h = the highest discernible f component in input signal

$f_{s(min)}$ = minimum sampling f

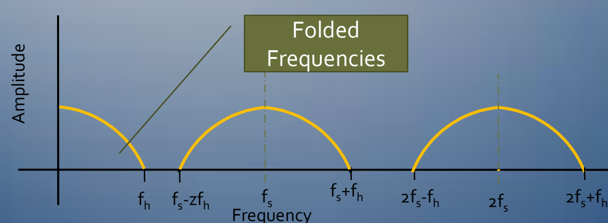
Nyquist rate is the minimum frequency and requires an ideal pulse to reconstruct the original signal into an analog value

Sampled Signal Frequency Spectrum

Sampling with $f_s > 2f_h$



Sampling at less than $2f_h$ causes aliasing and folding of sampled signals.



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Nyquist Frequency and Aliasing

Only signals with frequencies below Nyquist frequency will be correctly reproduced

Example: Given the following signal, determine the minimum sampling rate (Nyquist frequency)

$$s(t) = 2\cdot\sin(2\pi 100t) + 5\cdot\sin(2\pi 250t) + 1.5\cos(2\pi 500t) + 3\cdot\sin(2\pi 400t)$$

Find the highest frequency component: 100 Hz, 250 Hz, 500 Hz, 400 Hz

f_h= 500 Hz

$f_{s(min)} = 2f_h$

$f_{s(min)}$ = 2(500 Hz)= 1000 Hz

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Nyquist Frequency and Aliasing

Example: Given the following signal, determine the minimum sampling rate (Nyquist frequency)

$$s(t) = 1.5 \cdot \sin(175\pi t) + 3 \cdot \sin(250\pi t) + 0.5\cos(800\pi t) + 1.75 \cdot \sin(900\pi t)$$

Convert the radian frequency to frequency in Hz by dividing values by 2π

$$f_1 = \frac{175\pi}{2\pi} = 87.5\text{ Hz} \quad f_2 = \frac{250\pi}{2\pi} = 125\text{ Hz} \quad f_3 = \frac{800\pi}{2\pi} = 400\text{ Hz} \quad f_4 = \frac{900\pi}{2\pi} = 450\text{ Hz}$$

Find the highest frequency component: 450 Hz

$f_{s(min)} = 2f_h$

$f_{s(min)} = 2(450\text{ Hz}) = 900\text{Hz}$

Aliased Frequencies

Sampling analog signal below $2f_h$ produces false frequencies. Aliased frequencies determined by:

$$f_{alias} = |f_I - n \cdot f_s|$$

$$0 \le f_{alias} \le f_{nyquist}$$

$$f_{nyquist} = \frac{f_s}{2}$$

Where: f_I = sampled information signal with $f_I > f_{nyquist}$
f_s = sampling frequency (Hz)
n = sampling harmonic number
f_{alias} = aliased frequency
$f_{nyquist}$ = one-half sampling frequency

Samples/Period and Aliasing

Correct signal representation requires at least two samples/period

$$N_s = \frac{f_s}{f_I} = \frac{T_I}{T_s}$$

$$f_s > f_I \text{ and } T_I > T_s$$

Where N_s = number input signal samples per period of sampling frequency
f_s = sampling frequency (Hz)
f_I = highest information signal frequency (Hz)
T_s = sampling period, $1/f_s$, (seconds)
T_I = period information signal's highest frequency ($1/f_I$)

Sampling/Aliasing Examples

Example 1: A f_s=1000 Hz sampling frequency samples an information signal of f_I=100 Hz . Determine samples/period, the resulting recovered signal ,and aliased frequencies if present

Determine the number of samples/ period

$$N_s = \frac{1000\text{ Hz}}{100\text{ Hz}} = \frac{0.01\text{S}}{0.001\text{S}} = 10\text{ samples/period}$$

Above Nyquist rate of 2

$$f_{nyquist} = \frac{f_s}{2} = \frac{1000\text{ Hz}}{2} = 500\text{ Hz}$$

Signals below 500 Hz reproduced without aliasing

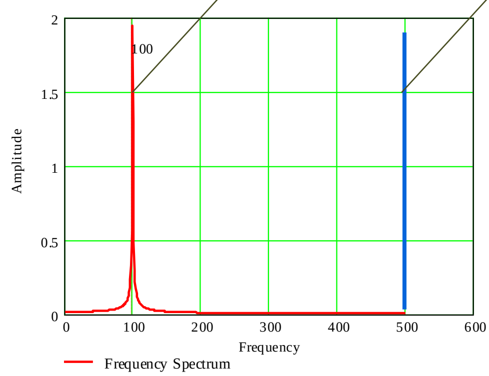
View the frequency spectrum using FFT of samples

Sampling/Aliasing Examples

Frequency Spectrum

100 Hz Recovered

500 Hz Nyquist Limit



Sampling/Aliasing Examples

Example 2: A f_s=60 Hz sampling frequency samples an information signal of f_I=100 Hz . Determine samples/period, the resulting recovered signal ,and aliased frequencies if present

Determine the number of samples/ period

$$N_s = \frac{60\ \text{Hz}}{100\ \text{Hz}} = \frac{0.01\ \text{S}}{0.001666\ \text{S}} = 0.6\ \text{samples/period}$$

Below Nyquist rate of 2

Aliased signals will occur due to low sampling rate

$$f_{nyquist} = \frac{f_s}{2} = \frac{60\ \text{Hz}}{2} = 30\ \text{Hz}$$

Signals below 30 Hz reproduced without aliasing

Now compute the aliased frequency for 1st sampling harmonic

Sampling/Aliasing Examples

Alias frequencies for 1st harmonic of sampling f (n=1)

$$f_{alias} = |f_I - n \cdot f_s|$$
$$0 \le f_{alias} \le f_{nyquist}$$
$$f_{nyquist} = \frac{f_s}{2}$$

$$f_{alias} = |100\ \text{Hz} - 1 \cdot 60\ \text{Hz}| = 40\ \text{Hz}$$
$$f_{nyquist} = \frac{60\ \text{Hz}}{2} = 30\ \text{Hz}$$
$$0 \le f_{alias} \le 30\ \text{Hz}$$

The f_{alias} is outside range 0-30 Hz, (40 Hz > 30 Hz) No recovered signal

Find alias frequencies of 2nd sampling harmonic f (n=2)

$$f_{alias} = |100\ \text{Hz} - 2 \cdot 60\ \text{Hz}| = 20\ \text{Hz}$$

The f_{alias} in range 0-30 Hz, 20 Hz recovered signal

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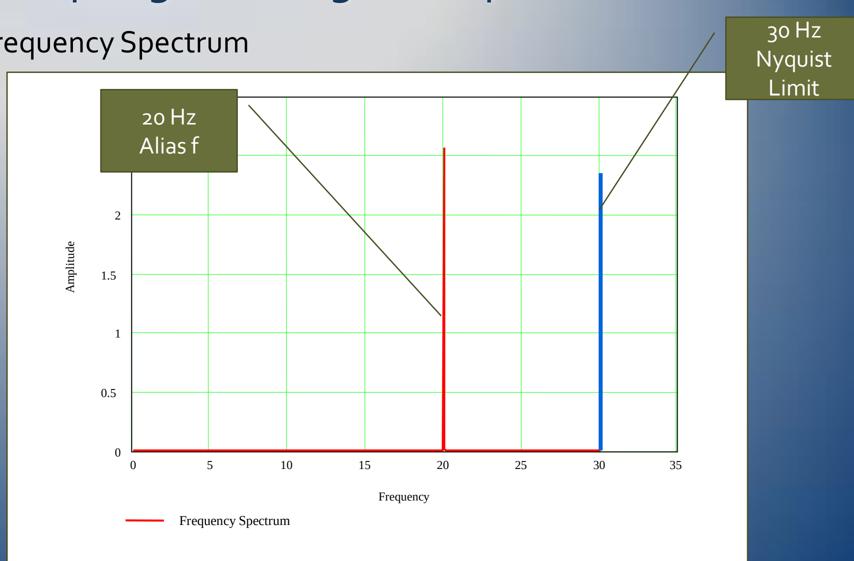
Sampling/Aliasing Examples

Frequency Spectrum



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Sampling/Aliasing Examples

Example 3: A f_s=80 Hz sampling frequency samples an information signal of f_I=100 Hz . Determine samples/period, the resulting recovered signal ,and aliased frequencies if present

Determine the number of samples/ period

$$N_s = \frac{80\ \text{Hz}}{100\ \text{Hz}} = \frac{0.01\ \text{S}}{0.00125\ \text{S}} = 0.8\ \text{samples/period}$$

Below Nyquist rate of 2

Aliased signals will occur due to low sampling rate

$$f_{nyquist} = \frac{f_s}{2} = \frac{80\ \text{Hz}}{2} = 40\ \text{Hz}$$

Signals below 40 Hz reproduced without aliasing

Sampling/Aliasing Examples

Alias frequencies for 1st harmonic of sampling f (n=1)

$$f_{alias} = |f_I - n \cdot f_s|$$
$$0 \le f_{alias} \le f_{nyquist}$$
$$f_{nyquist} = \frac{f_s}{2}$$

$$f_{alias} = |100\ \text{Hz} - 1 \cdot 80\ \text{Hz}| = 20\ \text{Hz}$$
$$f_{nyquist} = \frac{60\ \text{Hz}}{2} = 40\ \text{Hz}$$
$$0 \le f_{alias} \le 40\ \text{Hz}$$

The f_{alias} is inside range 0-40 Hz
20 Hz recovered signal

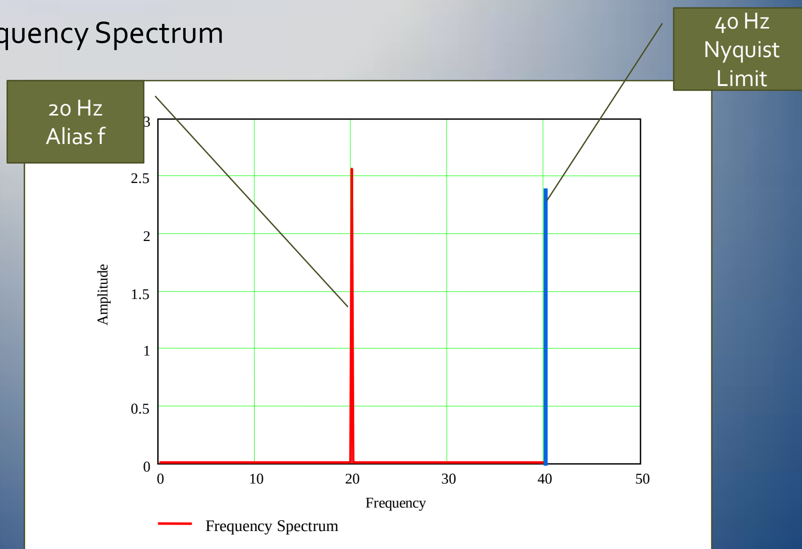
Find alias frequencies of 2nd sampling harmonic f (n=2)

$$f_{alias} = |100\ \text{Hz} - 2 \cdot 80\ \text{Hz}| = 60\ \text{Hz}$$

The f_{alias} outside range 0-40 Hz, No recovered signal

Sampling/Aliasing Examples

Frequency Spectrum



Sampling/Aliasing Examples

Example 4: A f_s=100 Hz sampling frequency samples an information signal of f_i=100 Hz . Determine samples/period, the resulting recovered signal ,and aliased frequencies if present

Determine the number of samples/ period

$$N_s = \frac{100\text{ Hz}}{100\text{ Hz}} = \frac{0.01\text{S}}{0.01\text{S}} = 1\text{ samples/period}$$

Below Nyquist rate of 2

Aliased signals will occur due to low sampling rate

$$f_{nyquist} = \frac{f_s}{2} = \frac{100\text{ Hz}}{2} = 50\text{ Hz}$$

Signals below 50 Hz reproduced without aliasing

Sampling and Aliasing Examples

Alias frequencies for 1^{st} harmonic of sampling f (n=1)

$$f_{alias} = |f_I - n \cdot f_s|$$
$$0 \le f_{alias} \le f_{nyquist}$$
$$f_{nyquist} = \frac{f_s}{2}$$

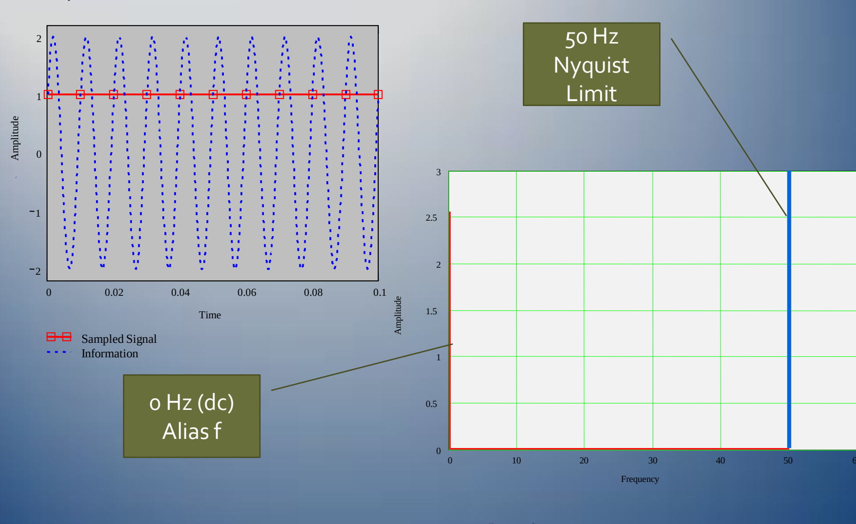
$$f_{alias} = |100\ \text{Hz} - 1 \cdot 100\ \text{Hz}| = 0\ \text{Hz}$$
$$f_{nyquist} = \frac{100\ \text{Hz}}{2} = 50\ \text{Hz}$$
$$0 \le f_{alias} \le 50\ \text{Hz}$$

The f_{alias} is inside range 0-50 Hz. 0 Hz indicates that the recovered signal is a dc level

View time and frequency plots of this example. 0 Hz is dc. Level depends on phase shift of information signal relative to sampling signal

Sampling and Aliasing Examples

Time plot



Sampling and Aliasing Examples

Previous examples all demonstrate under-sampling. $f_s \le f_I$ Folding occurs when $f_s > f_I$ but less that $f_{nyquist}$

Example 5: A f_s=125 Hz sampling frequency samples an information signal of f_I=100 Hz . Determine samples/period, the resulting recovered signal ,and aliased frequencies if present

Determine the number of samples/ period

$$N_s = \frac{125\ \text{Hz}}{100\ \text{Hz}} = \frac{0.01\ \text{S}}{0.008\ \text{S}} = 1.25\ \text{samples/period}$$

Below Nyquist rate of 2

Aliased signals will occur due to low sampling rate

$$f_{nyquist} = \frac{f_s}{2} = \frac{125\ \text{Hz}}{2} = 62.5\ \text{Hz}$$

Signals below 62.5 Hz reproduced without aliasing

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Sampling and Aliasing Examples

Alias frequencies for 1st harmonic of sampling f (n=1)

$$f_{alias} = |f_I - n \cdot f_s|$$
$$0 \le f_{alias} \le f_{nyquist}$$
$$f_{nyquist} = \frac{f_s}{2}$$

$$f_{alias} = |100\ \text{Hz} - 1 \cdot 125\ \text{Hz}| = 25\ \text{Hz}$$
$$f_{nyquist} = \frac{125\ \text{Hz}}{2} = 62.5\ \text{Hz}$$
$$0 \le f_{alias} \le 62.5\ \text{Hz}$$

The f_{alias} is inside range 0-62.5 Hz. A 25 Hz signal is reconstructed

Find alias frequencies of 2nd sampling harmonic f (n=2)

$$f_{alias} = |100\ \text{Hz} - 2 \cdot 125\ \text{Hz}| = 150\ \text{Hz}$$

The f_{alias} outside range 0-62.5 Hz, No recovered signal at this frequency

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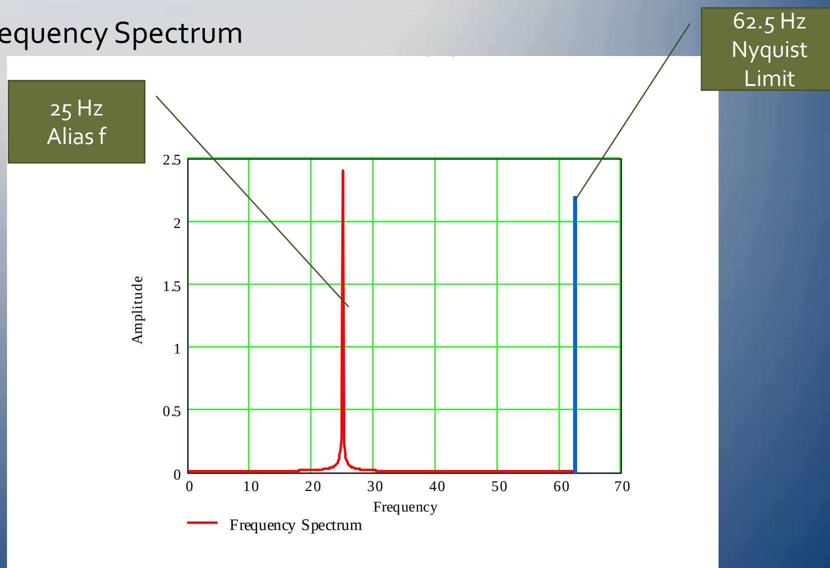
Sampling and Aliasing Examples

Frequency Spectrum



End Lesson 6: Sampling Analog Signals

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