Lesson 5: Power In Balanced Three-Phase Systems

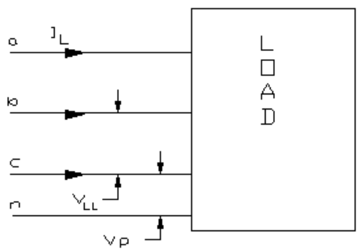
ET 332a Ac Motors, Generators and Power Systems

Learning Objectives

After this presentation you will be able to:

- Compute the active power absorbed by a three-phase load.
- Compute the reactive power absorbed by a three-phase load.
- Compute total three-phase apparent power.
- Compute the total power several loads absorb.
- Compute reactive power necessary to correct power factor in a three-phase system.

Three Phase Power



Load can be either wye or delta connected.

Assume wye connected load and compute power on per phase basis

Phase Power

$$P_\phi = V_p \cdot I_p \cdot \cos(\theta)$$
$$S_\phi = V_p \cdot I_p$$

Total Three-phase Power

$$P_T = 3 \cdot V_p \cdot I_p \cdot \cos(\theta)$$
$$S_T = 3 \cdot V_p \cdot I_p$$

Using line quantities...for wye connected loads

$$I_L = I_p$$
$$V_{LL} = \sqrt{3} \cdot V_p \rightarrow V_p = \frac{V_{LL}}{\sqrt{3}}$$

Three Phase Power

Total three-phase power using line quantities

Active Power $P_T = \sqrt{3} \cdot V_{LL} \cdot I_L \cdot \cos(\theta)$

Where θ is the angle between line voltage and line current

Apparent Power $S_T = \sqrt{3} \cdot V_{LL} \cdot I_L$

Three-phase reactive power formulas

Phase Reactive $Q_p = V_p \cdot I_p \cdot \sin(\theta_p)$

Where θ_p is the angle between phase voltage and phase current

Total Reactive $Q_T = 3 \cdot V_p \cdot I_p \cdot \sin(\theta)$

Also $S_T = \sqrt{P_T^{\ 2} + Q_T^{\ 2}}$ and $F_p = \frac{P_T}{S_T}$

3ϕ power factor

Three Phase Power Calculations

Example 5-1: A balanced delta connected three-phase load draws 200 A per phase with a leading power factor of 0.85 from a 12.47 kV line to line system. Determine the following :

a) The line current magnitude of the load
b) the phase voltage magnitude of the load
c) the total apparent power of the load
d) the total real power drawn by the load
e) the total reactive power drawn by the load
f) the total complex power of the load

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Example 5-1 Solution (1)

a) Delta connected load so $|V_L|=|V_p|$ and $|I_L|=\sqrt{3}\cdot|I_p|$

$I_L = \sqrt{3}\, I_p \qquad I_p = 200\,A \qquad I_L = \sqrt{3}(200A)$

$I_L = 346.4\ A$ Ans

b) Line voltage $V_L = V_p = 12.47\ kV$ Ans

c) Find the total apparent power, S_T $\quad S_T = \sqrt{3}\, V_L I_L$ OR $3 V_p I_p$

Using line quantities

$S_T = \sqrt{3}(12.47kV)(346.4A)$

$S_T = 7482\ kVA$ Ans

Using phase quantities

$S_T = 3 V_p I_p$

$S_T = 3(12.47kV)(200A)$

$S_T = 7482\ kVA$ Ans

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Example 5-1 Solution (2)

d) Active or real power

$P_T = \sqrt{3} V_L I_L \cos\theta$

Find θ from F_p

$F_p = 0.85$ Leading

$F_p = \cos\theta = 0.85$

$P_T = \sqrt{3}(12.47\text{kV})(346.4\text{A})(0.85)$

$P_T = 6359.5\ \text{KW}$ Ans

e) Reactive power $Q_T = \sqrt{3} V_L I_L \sin(\theta)$

Find θ

$\theta = \cos^{-1}(F_p)$

$\theta = \cos^{-1}(0.85)$

$\theta = 31.79^\circ$

$Q_T = \sqrt{3}(12.47\text{kV})(346.4\text{A})\sin(31.79^\circ)$

$Q_T = 3941\ \text{KVAR}$ Ans

Example 5-1 Solution (3)

f) Complex power. Assume V_L angle is zero degrees

$\bar{I}_L = 346.4\angle 31.79^\circ\ \text{A}$

$\bar{V}_L = 12.47\angle 0^\circ\ \text{KV}$

$$\bar{S}_T = \sqrt{3}\,\bar{V}_L\,\bar{I}_L^*$$

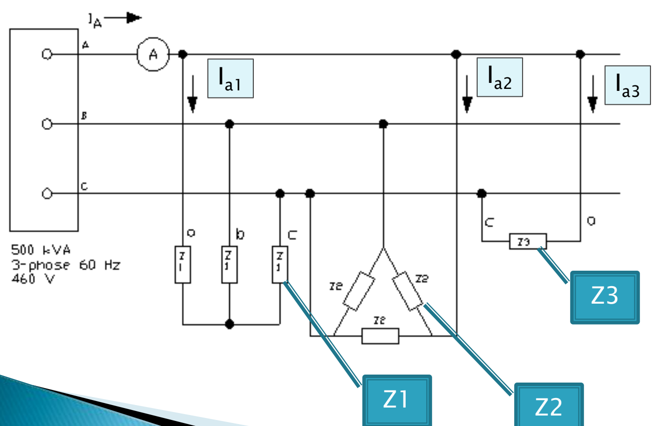
$$\bar{S}_T = \sqrt{3}(12.47\angle 0^\circ)(346.4\angle 31.79^\circ)^*$$

Polar form $\bar{S}_T = 7482\angle -31.79^\circ\ \text{KVA}$

Rectangular form $\bar{S}_T = \underbrace{6359.5}_{P} - j\underbrace{3941}_{Q}\ \text{KVA}$

Solving Three-phase Systems

Example 5-2: For the system below find the ammeter reading, I_A.
$E_{ab} = 460\angle 0^\circ$ V $E_{bc} = 460\angle -120^\circ$ V $E_{ca} = 460\angle 120^\circ$ V
$Z1 = 10\angle 30^\circ\ \Omega$ $Z2 = 15\angle 10^\circ\ \Omega$ $Z3 = 20+j20\ \Omega$



Example 5-2 Solution (1)

$\bar{E}_{ab} = 460\angle 0^\circ$ $\bar{Z}_1 = 10\angle 30^\circ\ \Omega$
$\bar{E}_{bc} = 460\angle -120^\circ$ $\bar{Z}_2 = 15\angle 10^\circ\ \Omega$
$\bar{E}_{ca} = 460\angle 120^\circ$ $\bar{Z}_3 = 20 + j20\ \Omega$

Meter reading is sum of load currents

$$\bar{I}_A = \bar{I}_{a1} + \bar{I}_{a2} + \bar{I}_{a3}$$

Z1 is a balanced wye connected load

$$\bar{I}_L = \bar{I}_P \quad \bar{I}_P = \frac{\bar{V}_P}{\bar{Z}_1} \quad \bar{V}_P = \frac{\bar{E}_{ab}}{\sqrt{3}}\angle 0 - 30^\circ$$

$$\bar{V}_P = \frac{460}{\sqrt{3}}\angle -30^\circ$$

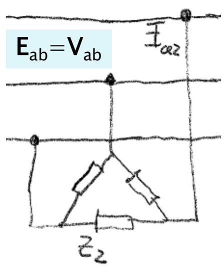
$$\bar{V}_P = 265.6\angle -30^\circ$$

Example 5-2 Solution (2)

$\bar{I}_L = \frac{265.6\angle -30^\circ}{10\angle 30^\circ}$ $\qquad$ $\bar{I}_L = 26.56\angle -60^\circ A$

$\bar{I}_{a1} = 26.56\angle -60^\circ A$

Z2 balanced delta load. Find I_p then compute I_L.



$\bar{I}_p = \frac{\bar{V}_{ab}}{\bar{Z}_2}$ $\qquad$ $\bar{V}_p = \bar{V}_L$ $\qquad$ $\bar{V}_{ab} = 460\angle 0^\circ V$

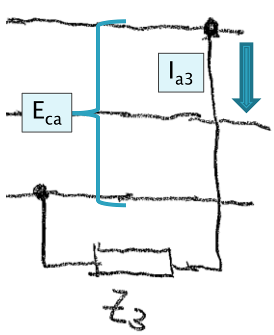
$$\bar{I}_p = \frac{460\angle 0^\circ V}{15\angle 10^\circ \Omega} = 30.67\angle -10^\circ A$$

$$\bar{I}_{a2} = \sqrt{3}(30.67)\angle -10-30^\circ A$$

$$\bar{I}_{a2} = 53.11\angle -40^\circ A$$

Example 5-2 Solution (3)

Find the single phase current



$\bar{I}_{a3} = \frac{\bar{E}_{ac}}{\bar{Z}_3}$ $\qquad$ $\bar{Z}_3 = 20 + j20\,\Omega$

$\bar{Z}_3 = 28.28\angle 45^\circ \Omega$

$\bar{E}_{ac} = -\bar{E}_{ca}$

$\bar{E}_{ca} = 460\angle 120^\circ V$

$\bar{E}_{ac} = -460\angle 120^\circ V$

$\bar{E}_{ac} = 1\angle 180^\circ (460\angle 120^\circ)$

$\bar{E}_{ac} = 460\angle 300^\circ = 460\angle -60^\circ$

$\bar{I}_{a3} = \frac{\bar{E}_{ac}}{\bar{Z}_3} = \frac{460\angle -60^\circ}{28.28\angle 45^\circ}$

$\bar{I}_{a3} = 16.27\angle -105^\circ A$

Example 5-2 Solution (4)

Sum currents to find meter reading

$$\bar{I}_A = 26.26\angle{-60^\circ} + 53.11\angle{-40^\circ} + 16.27\angle{-105^\circ}\ \text{A}$$

$$\bar{I}_A = (13.28 - j23) + (40.685 - j34.13) + (-4.21 - j15.72)$$

$$\bar{I}_A = 49.755 - j72.85\ \text{A}$$

$$\bar{I}_A = 88.22\angle{-55.67^\circ}\ \text{A}$$ Ans

Solving Three-Phase Systems by Power Calculations

<u>Example 5-3:</u> A 440 V 60 Hz 3-phase source supplies three loads:

1) Delta connected 3-phase 60 hp induction motor operating at 3/4 of rated output with an efficiency of 90% and a power factor of 94%
2) A wye connected 3-phase 75 hp induction motor operating at half of its rated output with an efficiency of 88% and a power factor of 74%
3) A delta connect resistive heater drawing 20 kW.

<u>Find:</u>

a) total active, reactive, and apparent power supplied by the source
b) the power factor of the combined loads
c) the magnitude of the line current
d) the capacitance and voltage rating for a wye connected capacitor bank that will correct the system power factor to 0.95 lagging

Solution Method

Find the total active and reactive power absorbed by each load, using the power factor and the efficiency, then construct power triangle for total load.

Use the following formulas

$$P_T = \sqrt{3}\cdot V_{LL}\cdot I_L\cdot\cos(\theta) \qquad S_T = \sqrt{P_T^{\ 2}+Q_T^{\ 2}} \qquad F_p = \frac{P_T}{S_T} = \cos(\theta) \qquad \eta = \left(\frac{P_0}{P_i}\right)\cdot 100\%$$

$$LF = \left(\frac{P_0}{P_{rated}}\right)\cdot 100\% \qquad \tan(\theta) = \frac{Q_T}{P_T}$$

Where: P_o = machine mechanical shaft output (W)
P_i = machine electrical input (W)
P_{rated} = machine rated shaft power (W)
P_T = total 3-phase active power (W)
S_T = total 3-phase apparent power (VA)
Q_T = total 3-phase reactive power (VARs)
η = machine efficiency
LF = load factor

Example 5-3 Solution (1)

a) Load 1: 60 Hp induction motor

$\eta_1 = 90\%$ $LF_1 = 75\%$ $F_{p1} = 94\% = 0.94$ $P_{r1} = 60\ HP$

$$P_{i1} = \frac{(746\,W/HP)\,P_{r1}\,(LF/100)}{\eta_1/100}$$

$$P_{i1} = \frac{(746\,W/HP)(60\,HP)(75/100)}{(90\%/100)}$$

$$P_{i1} = \frac{33,570\,W}{0.90} = 37,300\,W$$

Find power factor angle

$$\theta_1 = \cos^{-1}(F_{p1})$$

$$\theta_1 = \cos^{-1}(0.94)$$

$$\theta_1 = 19.95^\circ$$

Induction motor-F_p lagging Q_T positive

Example 5-3 Solution (2)

Find Q_{T1} $\tan(\theta) = \frac{Q_{T1}}{P_{i1}}$

$$P_{i1}\tan(\theta) = Q_{T1}$$

$$37{,}300 \tan(19.95^\circ) = Q_{T1}$$

$$\text{VARS} \quad 13{,}539 = Q_{T1}$$

Load 2: 75 hp induction motor

$$\eta_2 = 88\% \quad F_{p2} = 0.74$$

$$P_{r2} = 75\,\text{Hp} \quad LF = 50\%$$

$$P_{i2} = \frac{(746\,\text{W/HP})(P_{i2})(LF/100)}{(\eta_2/100)}$$

$$P_{i2} = \frac{(746\,\text{W/HP})(75\,\text{HP})(\frac{50\%}{100})}{(88\%/100)}$$

$$P_{i2} = \frac{27975\,\text{W}}{0.88} = 31{,}790\,\text{W}$$

Example 5-3 Solution (3)

Find the power factor angle then find Q_{T2}

$$\theta_2 = \cos^{-1}(F_{p2}) \qquad P_{i2}\tan(\theta_2) = Q_{T2}$$

$$\theta_2 = \cos^{-1}(0.74) \qquad 31{,}790 \tan(42.3^\circ) = Q_{T2}$$

$$\theta_2 = 42.3^\circ \qquad Q_{T2} = 28895\ \text{VARS}$$

Load 3; Resistance heater $P_{i3} = 20{,}000\,\text{W}$ Only absorbs watts

Sum the total active and reactive power of each load to find the total system power absorbed.

Example 5-3 Solution (4)

$P_{i1} = 37{,}300\ W$

$P_{i2} = 31{,}790\ W$

$P_{i3} = 20{,}000\ W$

$P_{sys} = 89{,}090\ W$ Ans

$Q_{T1} = 13{,}539\ VAR$

$Q_{T2} = 28{,}895\ VAR$

$Q_{sys} = 42{,}434\ VAR$ Ans

$$S_{sys} = \sqrt{P_{sys}^2 + Q_{sys}^2}$$

$$S_{sys} = \sqrt{89{,}090^2 + 42{,}434^2}$$

$$S_{sys} = 98{,}680\ VA$$ Ans

Example 5-3 Solution (5)

b) Find the power factor of the combined load

$$F_{psys} = \frac{P_{sys}}{S_{sys}} = \frac{89090\ W}{98{,}680\ VA}$$

$$F_{psys} = 0.903\ \text{Lag}$$ Ans

c) Find the load current magnitude

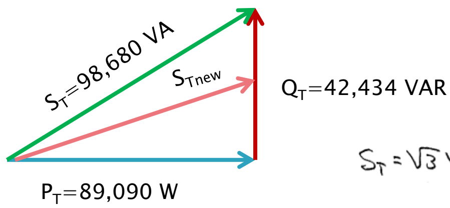
$$S_{sys} = \sqrt{3}\, V_L I_L \qquad V_L = 440\ V$$

$$I_L = \frac{S_{sys}}{\sqrt{3}\, V_L} \qquad I_L = \frac{98{,}680\ VA}{\sqrt{3}\ 440\ V}$$

$$I_L = 129.5\ A$$ Ans

Example 5-3 Solution (6)

d) Find the capacitance and voltage rating of a capacitor that corrects the system power factor to 0.95 lagging.



$Q_T = \sqrt{3} V_L I_L \sin\theta$

Where θ is system power factor angle

$S_T = \sqrt{3} V_L I_L \quad S_T = \frac{P_T}{F_p} \quad F_{pNew} = 0.95$

$\theta_{New} = \cos^{-1}(F_{pNew})$

$\theta_{New} = 18.2°$

Construct new triangle

$S_{TNew} = \frac{P_{sys}}{F_{pNew}} = \frac{89{,}040\,W}{0.95} \quad S_{TNew} = 93{,}726\ VA$

Example 5-3 Solution (7)

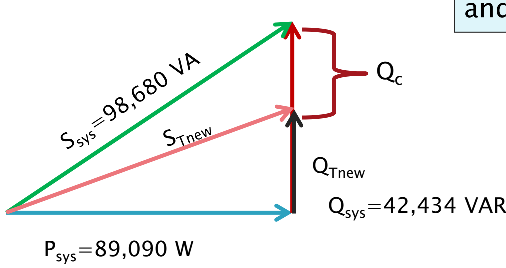
$Q_{TNew} = S_{TNew} \sin\theta_{New}$

$Q_{TNew} = 93{,}726 \sin(18.2°)$

$Q_{TNew} = 29{,}274\ VARS$

This is the value of the total 3-phase VARs required to correct F_p to 0.95 lagging

The capacitor bank power, Q_c, is the difference between Q_{sys} and Q_{Tnew}



$Q_{sys} = 42{,}434$

$Q_{TNew} = 29{,}274\ VARS$

$Q_C = 42{,}434 - 29{,}274\ VARS$

$Q_C = 13{,}160\ VARS$

Example 5-3 Solution (8)

Use the one-step formula to check result

$$Q_c = P_{sys}\left[\tan\left(\cos^{-1}(F_p)\right) - \tan\left(\cos^{-1}(F_{pNew})\right)\right]$$

$F_p = 0.9 \quad F_{pNew} = 0.95 \quad P_{sys} = 89{,}090\ \text{W}$

$$Q_c = 89{,}090\left[\tan\left(\cos^{-1}(0.9)\right) - \tan\left(\cos^{-1}(0.95)\right)\right]$$

$$Q_c = 89{,}090\left[\tan(25.84^\circ) - \tan(18.2^\circ)\right]$$

$$Q_c = 89{,}090\left[0.4843 - 0.3287\right]$$

$Q_c = 13{,}855\ \text{VARs}$ Compares to previous

Example 5-3 Solution (9)

Q_c is the total bank power required, find the phase power

$Q_c = 3Q_p \quad Q_p = \dfrac{Q_c}{3} \quad Q_p = \dfrac{13{,}160\ \text{VARs}}{3} = 4387\ \text{VARs}$

Bank is wye connected so capacitors see V_{pn} not V_L

$V_{pN} = \dfrac{V_L}{\sqrt{3}} = \dfrac{440\text{V}}{\sqrt{3}} \quad V_{pN} = 254\text{V}$ Ans

Find the capacitor value

$C = \dfrac{Q_P}{2\pi f V_{pN}^2} \quad f = 60\text{Hz} \quad V_{pN} = 254\text{V} \quad Q_p = 4387\ \text{VAR} \quad C = \dfrac{4387\ \text{VAR}}{2\pi(60)(254)^2}$

$C = 0.00018\ \text{F} \quad C = \dfrac{0.00018}{1\times10^{-6}\ \text{F}/\mu\text{F}} = 180\ \mu\text{F}$ Ans

End Lesson 5: Power In Balanced Three-Phase Systems

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