Lesson 8: Induced Voltage in Practical dc Machines

ET 332a

Dc Motors, Generators and Energy Conversion Devices

Learning Objectives

After this presentation you will be able to:

- Compute the average output voltage of a dc generator given machine physical construction parameters.
- Draw the schematic circuit model of a dc machine
- Find generator output voltage using a magnetization curve and defining formulas.
- Compute generator voltage regulation.

Average Induced Voltage in a Generator

Start with fundamental equation

$$e = B \cdot l \cdot v \cdot \sin(\theta)$$

Where v = velocity of conductor
For rotating system

v = angular velocity $d\theta/dt = \omega$ rad/s

Angular velocity relates to frequency f (cycles/s or Hertz) by: $\omega = 2\pi f$

Frequency of e related to number of field poles and rotational speed by:

$$f = \frac{P \cdot n}{120}$$

Where: P = number of field poles
n = rotational speed (rpm)

Average Induced Voltage in a Generator

Substituting into fundamental equation and simplifying gives:

$$E_a = \frac{n \cdot P \cdot N_a \cdot \Phi_p}{30}$$

Where: P = number of poles
n = rotational speed
N_a = # of turns in coil
Φ_p = field flux
E_a = average induced voltage

Consider machine construction details and define another voltage formula

Average Induced Voltage in a Generator

Define the number of conductors in field in terms of windings

$$N_a = \frac{z_a}{2}$$

z_a = Total number of armature conductors in the field.

Substitute into previous equation to get:

$$E_a = \frac{n \cdot P \cdot z_a \cdot \Phi_p}{60 \cdot a}$$

Where:

a = number of parallel paths

Average Induced Voltage in a Generator

The number of poles, parallel paths, and conductors are constant once the machine is constructed so define:

$$k_G = \frac{P \cdot z_a}{60 \cdot a}$$

Where k_G = emf constant

The constant k_G also known as k_e in some texts. Relates motor voltage to speed and field flux.

$$E_a = k_G \cdot n \cdot \Phi_p = k_e \cdot n \cdot \Phi_p$$

Note : E_a is proportional to the flux and to the rotational speed. If conditions of machine are know at one n and Φ_p, another operating point can be found by equating ratios.

Induced Voltage Examples

Example 1: A 2 pole dc motor armature rotates at 1800 rpm. It has 400 turns in the armature winding. The magnetic field flux is 0.0025 Wb. Compute the average induced voltage.

$$E_a = \frac{n P N_c}{30}\cdot\Phi_P = \frac{(1800)(2)(400)(0.0025)}{30} = 120\ \text{Vdc}$$

average

Example 2: A 4 pole dc machine rotates at 200 rad/s in a magnetic field of 0.0048 Wb. There are 4 parallel current paths that have 200 conductors. Find the induced voltage in the armature and the emf constant for the machine.

Convert ω to n for formula

$$n = \left(\frac{60}{2\pi}\right)\cdot 200\ \text{rad/s} = 1910.8\ \text{rpm}$$

$$E_a = \frac{n\cdot P\cdot z_a\cdot \Phi_p}{60\cdot a} = \frac{(1910.8\ \text{rpm})(4)(200)(0.0048\ \text{Wb})}{60(4)}\boxed{= 30.57\ \text{Vdc}}$$

Induced Voltage Examples

Example 2 Continued: Find the emf constant

Lump the constants and compute

$$k_e = \frac{P\cdot z_a}{60\cdot a} = \frac{(4)(200)}{60(4)} = \boxed{3.333\ \text{V/Wb - rpm}}$$

Example 3: A 4 pole 50 kW dc machine has a value of E_a = 110 V at 1100 rpm. What is the induced voltage if the speed is increased 20%?

$E_{a1} = k_e n_1 \Phi_{p1}$ $\Phi_p = \Phi_{p1} = \Phi_{p2}$ No change in flux $E_{a1} = 110\ V$ $n_1 = 1100$ rpm

$E_{a2} = k_e n_2 \Phi_{p2}$

$n_2 = n_1 + 0.2(n_1)$

$n_2 = 1100 + 220 = 1320$ rpm

Find E_{a2} $\frac{E_{a1}}{E_{a2}} = \frac{k_e n_1 \Phi_p}{k_e n_2 \Phi_p} \Rightarrow \frac{E_{a1}}{E_{a2}} = \frac{n_1}{n_2}$ $\frac{n_2}{n_1} E_{a1} = E_{a2} = \frac{1320}{1100}(110) = E_{a2}$

132 Vdc $= E_{a2}$ Ans

Induced Voltage Examples

Example 4: A 4 pole dc machine has a value of $E_a = 50$ V at 400 rpm. What is the value of E_a if the pole flux is doubled while the speed remains constant?

$$E_{a1} = K_e n_1 \Phi_{p1} \quad E_{a2} = K_e n_2 \Phi_{p2}$$

$$n = n_1 \quad n_2 = 400$$

$$\Phi_{p2} = 2\Phi_{p1}$$

$$\frac{E_{a1}}{E_{a2}} = \frac{K_e n_1 \Phi_{p1}}{K_e n_2 \Phi_{p2}} = \frac{K_e(400)\Phi_{p1}}{K_e(400)2\Phi_{p1}}$$

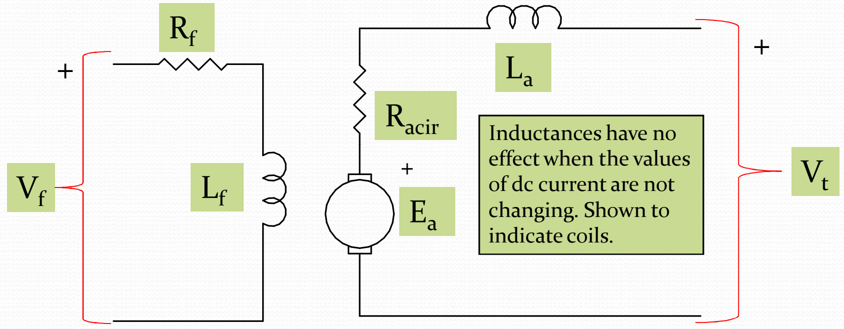
$$\frac{E_{a1}}{E_{a2}} = \frac{1}{2} \Rightarrow E_{a2} = 2E_{a1}$$

$$E_{a2} = 2(50) = 100 \text{ Vdc}$$

Ans

Assumes no saturation

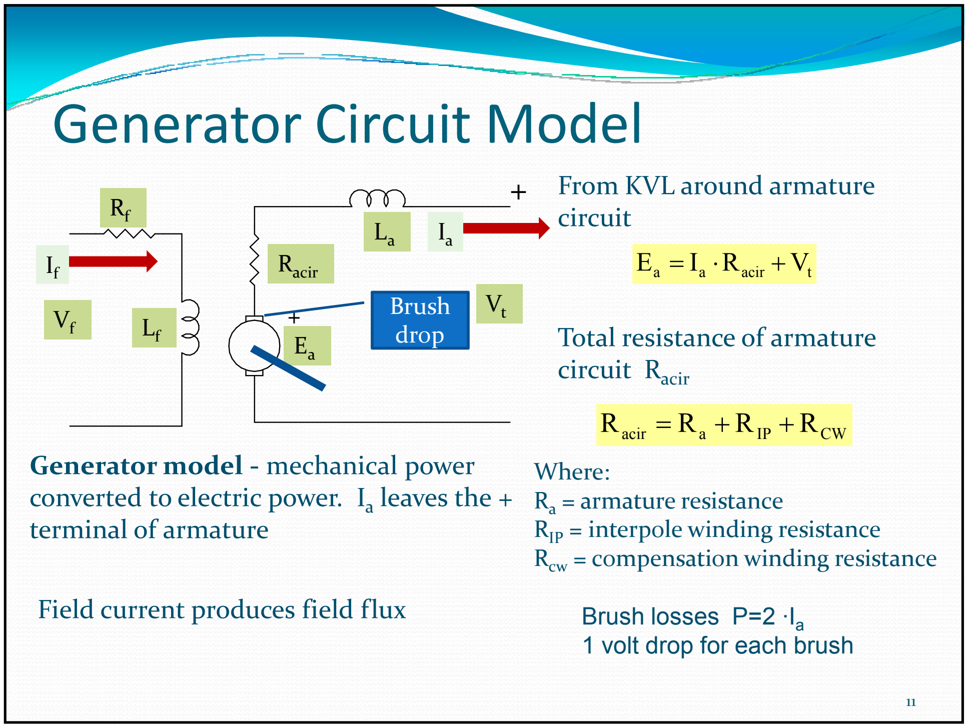
Circuit Model of Dc Machines



Inductances have no effect when the values of dc current are not changing. Shown to indicate coils.

E_a = internally induced voltage
V_t = terminal voltage of machine
V_f = field electromagnetic source voltage

R_{acir} = dc resistance of the armature windings , interpoles, etc
L_a = inductance of armature circuit
R_f = dc resistance of the field windings
L_f = inductance of the field windings



Generator Circuit Model

From KVL around armature circuit

$$E_a = I_a \cdot R_{acir} + V_t$$

Total resistance of armature circuit R_{acir}

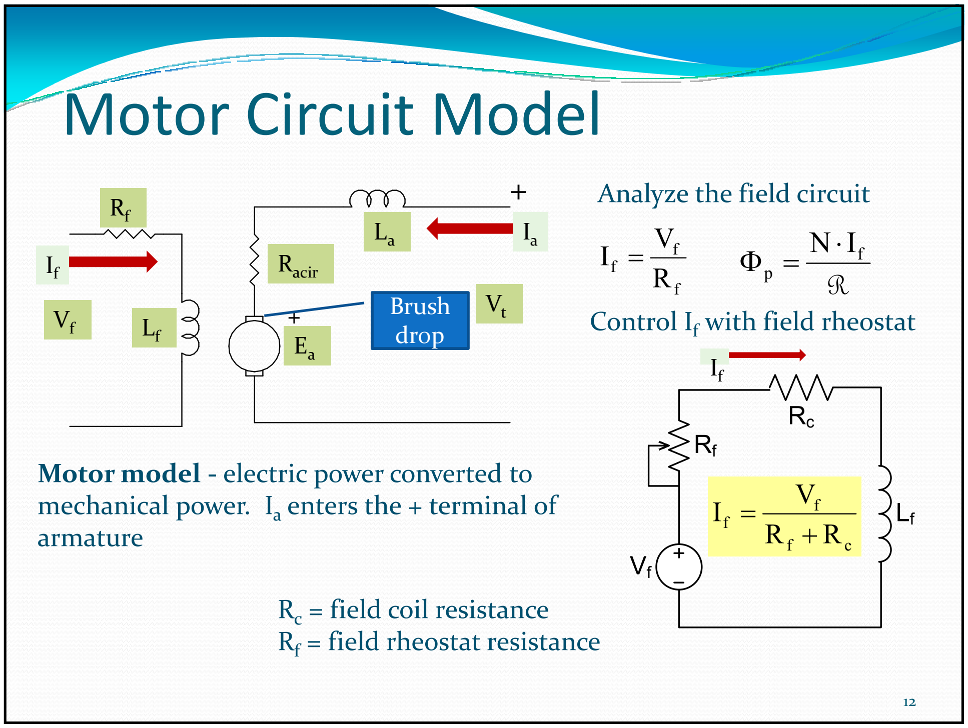
$$R_{acir} = R_a + R_{IP} + R_{CW}$$

Where:
R_a = armature resistance
R_{IP} = interpole winding resistance
R_{cw} = compensation winding resistance

Generator model - mechanical power converted to electric power. I_a leaves the + terminal of armature

Field current produces field flux

Brush losses $P=2 \cdot I_a$
1 volt drop for each brush



Motor Circuit Model

Analyze the field circuit

$$I_f = \frac{V_f}{R_f} \qquad \Phi_p = \frac{N \cdot I_f}{\mathfrak{R}}$$

Control I_f with field rheostat

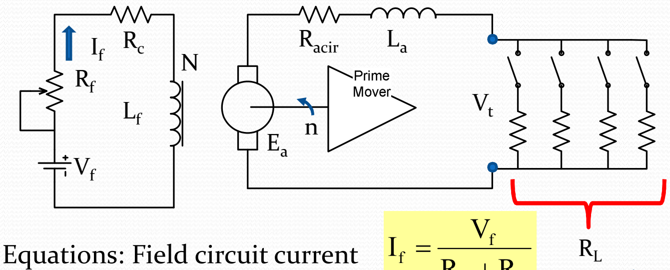
$$I_f = \frac{V_f}{R_f + R_c}$$

Motor model - electric power converted to mechanical power. I_a enters the + terminal of armature

R_c = field coil resistance
R_f = field rheostat resistance

Separately Excited Dc Generator

Separate source of supply develops field flux. Source called the **exciter**



Φ_p

Model Equations: Field circuit current $I_f = \frac{V_f}{R_f + R_c}$

Magnetic coupling to induced average voltage $E_a = \frac{N \cdot I_f \cdot n \cdot k_G}{\Re}$

E_a in terms of flux for constant I_f $E_a = k_G \cdot n \cdot \Phi_p$

$k_e = k_G \cdot \Phi_p$ $E_a = k_e \cdot n$

E_a proportional to n for constant I_f

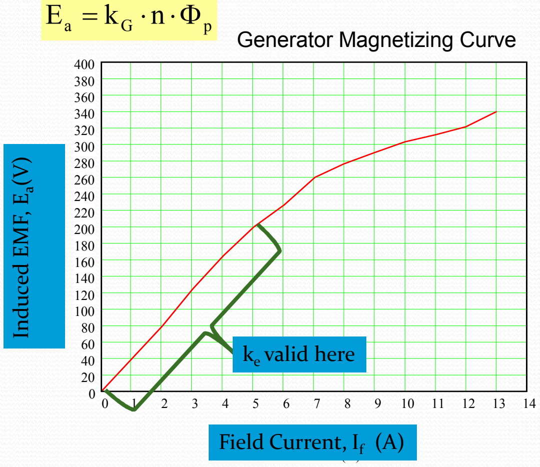
Separately Excited Dc Generator

Generally flux, Φ is non-linear with respect to the field current I_f.

$E_a = k_G \cdot n \cdot \Phi_p$

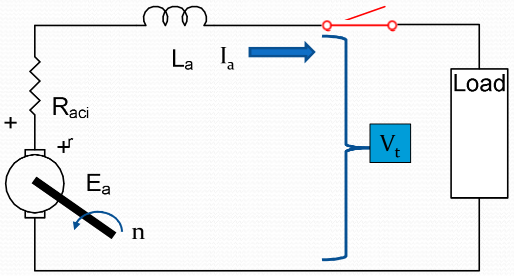
Magnetizing curve for generator gives relationship for E_a as a function of field current.

If I_f is in linear part of curve E_a is proportional to n and I_f.



Terminal Voltage Regulation

Voltage regulation finds the percent change in terminal voltage from no-load to full load



KVL around armature circuit with switch closed

$$E_a = I_a \cdot R_{acir} + V_t$$

With switch closed and n constant $V_t < E_a$ and decreases as I_a increases

$I_a = 0$ with switch open so:

$$E_a = 0 \cdot R_{acir} + V_t$$
$$E_a = V_t$$

No-load voltage = E_a

Terminal Voltage Regulation

For rated terminal voltage

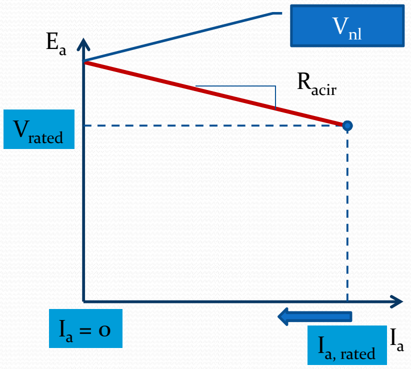
$$\%VR = \frac{V_{nl} - V_{rated}}{V_{rated}}$$

Where: %VR = percent voltage regulation
V_{nl} = no-load terminal voltage
V_{rated} = name plate rating of the terminal voltage when generator delivers rated power.

How does terminal voltage change with delivered power?

Terminal Voltage Regulation

Generator Terminal Voltage Vs Load Current



$E_a - I_a \cdot R_{acir} = V_t$

Equation above plots as downward sloping line for changes in armature current

As armature current decreases (load decreases)Terminal voltage increases (V_t increases)

%VR related to the armature circuit voltage drop. Smaller %VR is better.

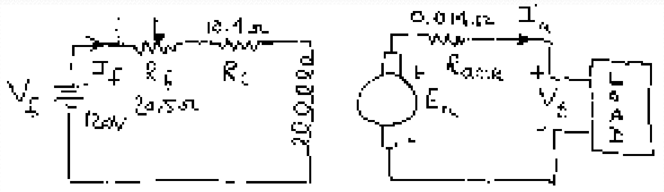
Voltage Regulation Example

Example 1 A separately excited 50 kW, 3500 rpm, 2-pole dc generator has a rated terminal voltage of 120 Vdc. Its exciter is supplied from a 120 Vdc supply. The field coil resistance is 10.4 Ω and the field rheostat is set at 20.5 Ω. The total armature resistance is 0.014 Ω. The generator is supplying 420 A to a load. The magnetizing curve is given on a previous slide.

Determine:
a.) the induced armature voltage at this level of excitation
b.) the terminal voltage at 100%, 75% and 50% load current

Voltage Regulation Solution

Draw schematic model for generator



$I_f = \frac{V_f}{R_f + R_c}$

$I_f = \frac{120 V}{20.5 + 18.4 \Omega}$

$I_f = 3.08 A$

Use I_f and magnetization curve to find E_a

From Magnetization Curve $E_a = 162 V$ Ans

b) $I_a = 420 A$ at 100% Load

KVL at terminal circuit $E_a - I_a R_{a,se} = V_t$

$E_a - I_a R_{a,se} = V_t$

$R_{a,se} = 0.014 \Omega$

Voltage Regulation Solution

Solution continued: Compute regulation at different load levels

ⓐ 100% Load

$162 - 420A(0.014 \Omega) = V_t$

$162 - 5.88 = V_t$

$V_t = 156.12 V$ Ans

ⓑ 75% Load

$I_{a75} = (75/100)(420 A) = 315 A$

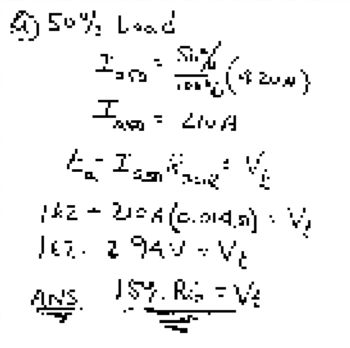
$E_a - I_{a75} R_{a,se} = V_t$

$162 - 315A(0.014\Omega) = V_t$

$V_t = 157.59$ Ans

Voltage Regulation Solution

Solution continued: Compute regulation at different load levels



4) 50% Load

$I_{a50} = \frac{50\%}{100\%}(420\,A)$

$I_{a50} = 210\,A$

$E_a - I_{a50} R_{acir} = V_t$

$162 - 210\,A(0.014\,\Omega) = V_t$

$162 - 2.94\,V = V_t$

Ans $159.86 = V_t$

Summary of calculations

% Load	I_a	V_t
100	420 A	156.12
75	315 A	157.59 V
50	210 A	159.86 V

Terminal voltage increases on generator as the load current decreases

Separately Excited Generator Example

Example 2: For the machine in Example 1, ,determine the field rheostat setting for the machine to deliver rated output current at rated voltage. Also determine the %VR at rated load.

Solution

Calculate the current at rated load for the generator power rating

$P_{rated} = 50\ kW = 50{,}000\ W$

$V_{rated} = V_t = 120\ V\ dc$

Find Rated I from power and voltage

$$I_{a,rated} = \frac{P_{rated}}{V_t} = \frac{50{,}000\ W}{120\ V} = 416.7\ A$$

Now find value of E_a with V_t = 120 V using $E_a = I_a \cdot R_{acir} + V_t$

Separately Excited Generator Example

Example 2 Continued

$$R_{acir} = 0.014\,\Omega$$
$$E_a = I_a \cdot R_{acir} + V_t$$
$$E_a = 416.7\text{ A} \cdot (0.014\,\Omega) + 120\text{ V}$$
$$E_a = 125.833\text{ V}$$

Use magnetization curve to find I_f

I_f = 2.8 A from graph. Now find R_f

$$I_f = \frac{V_f}{R_f + R_c}$$

Solve for R_f

$$I_f(R_f + R_c) = V_f$$
$$I_f R_f + I_f R_c = V_f$$

$\Rightarrow$

$$I_f R_f = V_f - I_f R_c$$
$$R_f = \frac{V_f}{I_f} - R_c$$

Separately Excited Generator Example

Example 2 Continued

$$R_f = \frac{V_f}{I_f} - R_c$$

From Previous example

$$R_c = 10.4\,\Omega$$

$$R_f = \frac{120\text{ V}}{2.8\text{ A}} - 10.4\,\Omega = \boxed{32.46\,\Omega}$$

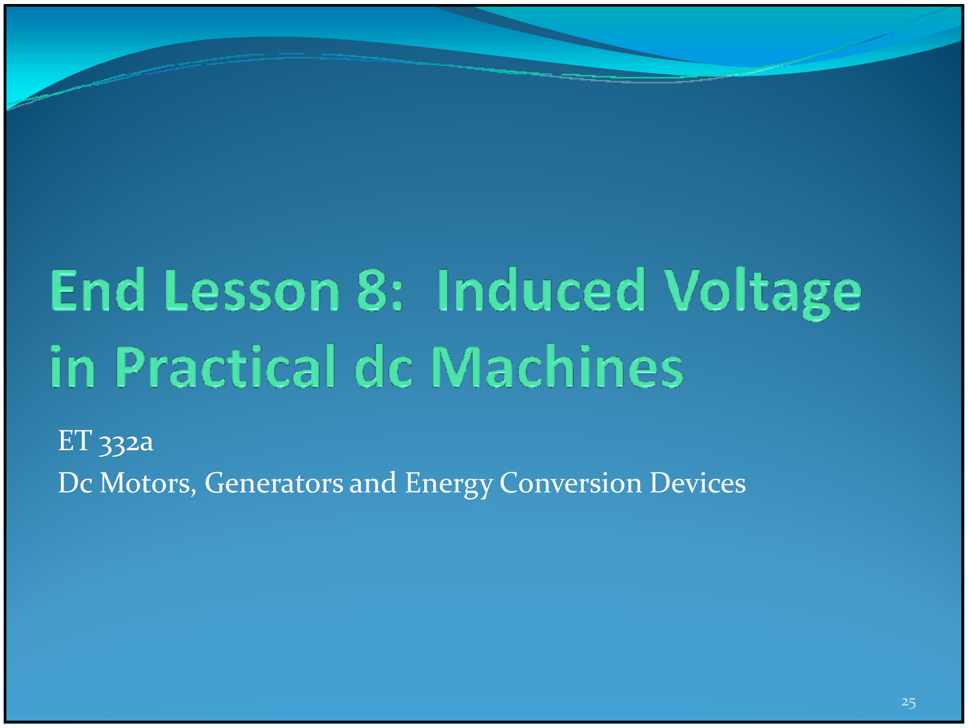
Find the % VR Percent Voltage Regulation $V_{rated} = V_t = 120\text{ V}$

$V_{nl} = V_t$ with I_a =0 $E_a = I_a \cdot R_{acir} + V_t$ $\Rightarrow$ $E_a = 0 \cdot R_{acir} + V_t = E_a = V_t$

V_{nl}=125.833

$$\%VR = \frac{V_{nl} - V_{rated}}{V_{rated}} \cdot 100\%$$

$$\%VR = \frac{125.833\text{ V} - 120\text{ V}}{120\text{ V}} \cdot 100\% = \boxed{4.86\%}$$



End Lesson 8: Induced Voltage in Practical dc Machines

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