4.4.4 ■ Culverts

Culverts are short conduits that are used to pass water under roads and highways. Culverts perform a similar function to that of bridges, but unlike bridges they have spans less than 6 m (20 ft) and can be designed to have a submerged inlet. Typical cross-sections of culverts include circular, arched, rectangular, and oval shapes.

Culvert design typically requires the selection of a barrel cross-section that passes a given flowrate when the water is ponded to a given height at the culvert entrance. The hydraulic analysis of culverts is complicated by the fact that there are several possible flow regimes, with the governing flow equation being determined by the flow regime. The flow regimes can be broadly grouped into *submerged-entrance conditions* and *free-entrance conditions*, which are illustrated in Figures 4.27 and 4.28, respectively. The

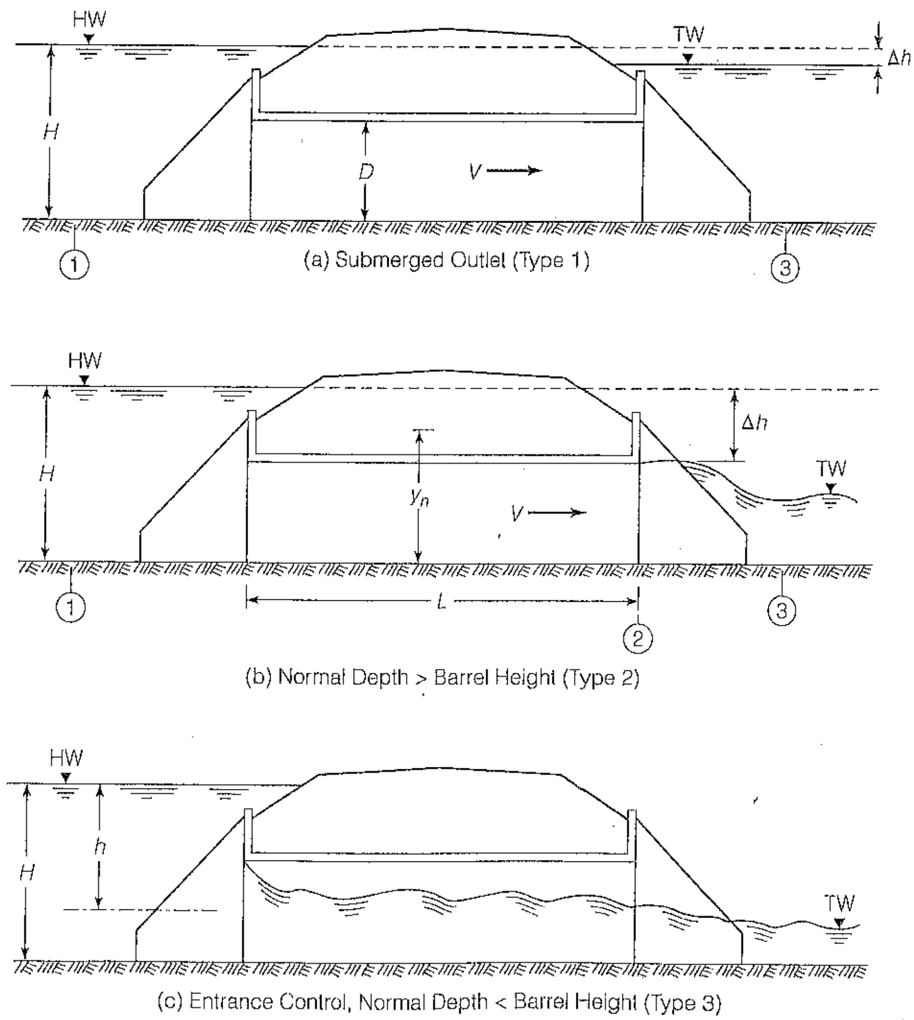

Figure 4.27 ■ Flow Through Culvert with Submerged Entrance
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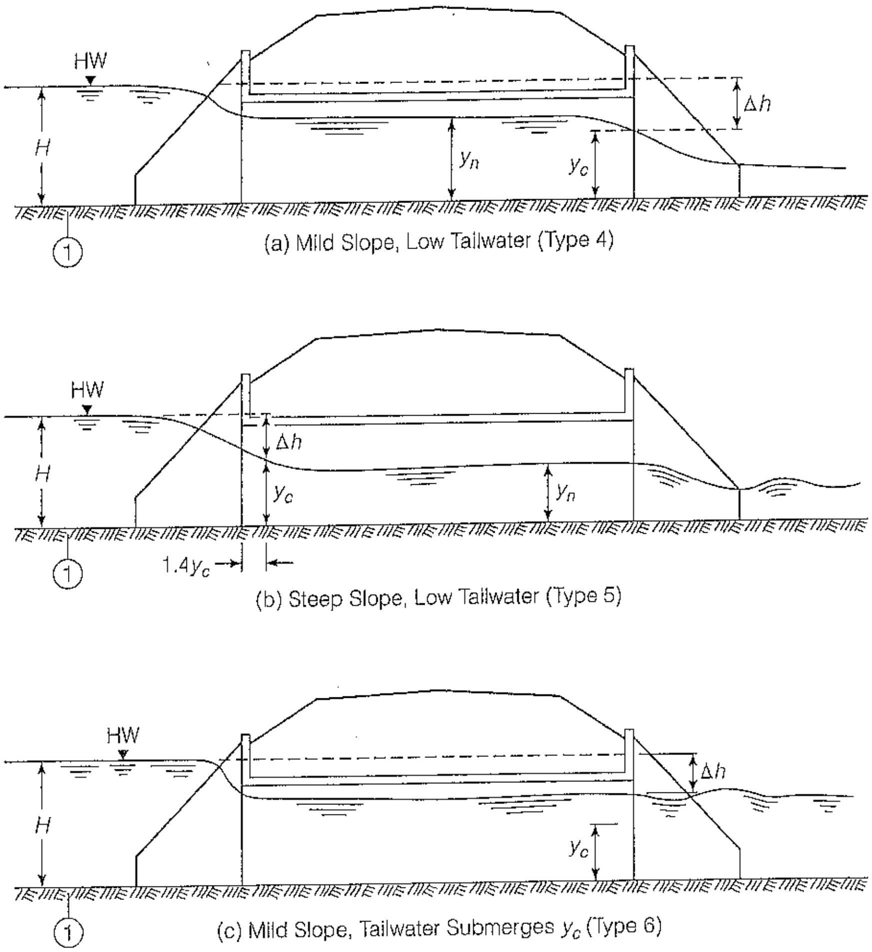


Figure 4.28 ■ Flow Through Culvert with Free Entrance

Source: Franzini, Joseph and Finnemore, E. John. *Fluid Mechanics*, 9e. Copyright © 1997 by The McGraw-Hill Companies. Reprinted by permission.

entrance to a culvert is regarded as submerged when the depth, H, of water upstream of the culvert exceeds $1.2D$, where D is the diameter of the culvert. In some references this limit is taken as $1.5D$ (e.g., French, 1985). In the case of a submerged entrance, Figure 4.27 indicates that there are three possible flow regimes: (a) the outlet is submerged (Type 1 flow); (b) the outlet is not submerged and the normal depth of flow in the culvert is larger than the culvert diameter, D (Type 2 flow); and (c) the outlet is not submerged and the normal depth of flow in the culvert is less than the culvert diameter (Type 3 flow).

In Type 1 flow, applying the energy equation between Sections 1 (headwater) and 3 (tailwater) leads to

$$\Delta h = h_i + h_f + h_o \tag{4.176}$$

where Δh is the difference between the headwater and tailwater elevations, h_i is the entrance loss, h_f is the head loss due to friction in the culvert, and h_o is the exit loss. Equation 4.176 neglects the velocity head at Sections 1 and 3, which is usually small compared with the other terms. Using the Manning equation to calculate h_f within the culvert, then

$$h_f = \frac{n^2V^2L}{R^{\frac{4}{3}}} \tag{4.177}$$

where n is the roughness coefficient, V is the velocity of flow, L is the length, and R is the hydraulic radius of the culvert. The entrance and exit losses, h_i and h_o, are given by

$$h_i = k_e \frac{V^2}{2g} \tag{4.178}$$

$$h_o = \frac{V^2}{2g} \tag{4.179}$$

where k_e is the entrance loss coefficient. Combining Equations 4.176 to 4.179 yields the following form of the energy equation between Sections 1 and 3

$$\Delta h = \frac{n^2 V^2 L}{R^{\frac{4}{3}}} + k_e \frac{V^2}{2g} + \frac{V^2}{2g} \tag{4.180}$$

This equation can also be applied between Section 1 (headwater) and Section 2 (culvert exit) in Type 2 flow, illustrated in Figure 4.27(b), where the velocity head at the exit, $V^2/2g$, is equal to the exit loss in Type 1 flow. Equation 4.180 reduces to the following relationship between the difference in the water-surface elevations on both sides of the culvert, Δh, and the discharge through the culvert, Q,

$$\boxed{Q = A\sqrt{\frac{2g\Delta h}{19.62n^2L/R^{\frac{4}{3}} + k_e + 1}}} \tag{4.181}$$

where A is the cross-sectional area of the culvert. It should be noted that Δh is equal to the difference between the headwater and tailwater elevations for a submerged outlet (Type 1 flow), and Δh is equal to the difference between the headwater and the crown of the culvert exit when the normal depth of flow in the culvert exceeds the height of the culvert (Type 2 flow).

Under both Type 1 and Type 2 conditions, the flow is said to be under *outlet control* since the water depth at the outlet influences the discharge through the culvert. In cases where the inlet is submerged and the culvert entrance will not admit water fast enough to fill the culvert (Type 3 flow in Figure 4.27), the culvert inlet behaves like an orifice and the discharge through the culvert, Q, is related to the head on the center of the orifice, h, by the relation

$$\boxed{Q = C_d A\sqrt{2gh}} \tag{4.182}$$

where C_d is the coefficient of discharge and h is equal to the vertical distance from the center of the culvert entrance to the water surface at the entrance. The coefficient of discharge, C_d, is equal to 0.62 for a square-edged entrance and approaches 1 for a well-rounded entrance (Franzini and Finnemore, 1997). In cases where the culvert entrance acts like an orifice, the downstream water level does not influence the flow through the culvert and the flow is said to be under *inlet control*. According to ASCE (1992), Equation 4.182 is applicable only when $H/D \geq 2$, but Franzini and Finnemore (1997) state that the error in Equation 4.182 is less than 2% when $H/D \geq 1.2$.

Submerged entrances usually lead to greater flows through the culvert than unsubmerged entrances. In some cases, however, culverts must be designed so that the entrances are not submerged. Such cases include those in which the top of the culvert forms the base of a roadway. In the case of an unsubmerged entrance, Figure 4.28 indicates that there are three possible flow regimes: (a) the culvert has a mild slope and a low tailwater, in which case the critical depth occurs somewhere near the exit of the culvert (Type 4 flow); (b) the culvert has a steep slope and a low tailwater, in which case the critical depth occurs somewhere near the entrance of the culvert, at approximately $1.4y_c$ downstream from the entrance (Type 5 flow); and (c) the culvert has a mild slope and the tailwater submerges y_c (Type 6 flow).

Applying the energy equation between the headwater and the culvert exit in Type 4 flow gives

$$\Delta h + \frac{V_1^2}{2g} - \frac{V^2}{2g} = h_i + h_f \tag{4.183}$$

where Δh is the difference between the headwater elevation and the elevation of the (critical) water surface at the exit of the culvert, V_1 is the headwater velocity, h_i is the entrance loss given by Equation 4.178, and h_f is the friction loss in the culvert given by Equation 4.177. The velocity of the headwater is *not* neglected, as in the case of a ponded headwater where $H/D > 1.2$. Equation 4.183 yields the following expression for the discharge, Q, through the culvert,

$$\boxed{Q = A_c\sqrt{2g(\Delta h + V_1^2/2g - h_i - h_f)}} \tag{4.184}$$

where A_c is the flow area at the critical flow section at the exit of the culvert. Equation 4.184 is actually an implicit expression for the discharge, since the entrance loss, h_i, and the friction loss, h_f, depend on the discharge, Q.

In Type 5 flow (steep slope, low tailwater), the critical flow depth occurs at the entrance of the culvert. Applying the energy equation between the headwater and the culvert entrance gives

$$\Delta h + \frac{V_1^2}{2g} - \frac{V^2}{2g} = h_i \tag{4.185}$$

where Δh is the difference between the headwater elevation and the elevation of the (critical) water surface at the entrance of the culvert. Equation 4.185 leads to the following expression for the discharge, Q, through the culvert

$$\boxed{Q = A_c\sqrt{2g(\Delta h + V_1^2/2g - h_i)}} \tag{4.186}$$

Finally, in Type 6 flow (mild slope, tailwater submerges y_c), the water surface at the culvert exit is approximately equal to the tailwater elevation. Applying the energy equation between the headwater and the culvert exit gives

$$\Delta h + \frac{V_1^2}{2g} - \frac{V^2}{2g} = h_i + h_f \tag{4.187}$$

where Δh is the difference between the headwater elevation and the tailwater elevation at the exit of the culvert. Equation 4.187 leads to the following expression for the discharge, Q, through the culvert,

$$\boxed{Q = A\sqrt{2g(\Delta h + V_1^2/2g - h_i - h_f)}} \tag{4.188}$$

where A is the flow area at the exit of the culvert.

Determination of Flow Type. The flow type is determined by the headwater, tailwater, and culvert dimensions. Based on the headwater elevation and culvert dimensions, it is determined whether the culvert entrance is submerged. If $H/D > 1.2$, the culvert entrance is submerged; if not, the entrance is not submerged.

Submerged Entrance. When the culvert entrance is submerged, the flow is either Type 1, 2, or 3. The flow type and the associated flowrate are determined by the following procedure:

1. If the culvert exit is submerged, then the flow is Type 1 and the discharge is given by Equation 4.181.
2. If the culvert exit is not submerged, the flow is either Type 2 or 3. Assume that the flow is Type 2 and calculate the discharge using Equation 4.181, with the appropriate definition of Δh.
3. Use the flowrate calculated in step 2 to determine the normal depth of flow in the culvert.
4. If the normal depth of flow calculated in step 3 is greater than the height of the culvert, then the flow is Type 2 and the discharge calculated in step 2 is correct.
5. If the normal depth of flow calculated in step 3 is less than the height of the culvert, then the flow is probably Type 3. Calculate the discharge using Equation 4.182 and verify that the normal depth of flow in the culvert is less than the culvert height. If the normal depth is less than the culvert height, then Type 3 flow is confirmed.
6. If neither Type 2 nor Type 3 flow can be confirmed, take the capacity of the culvert to be the lesser of the two calculated discharges.

It is useful to note that circular culverts flow full when the discharge rate exceeds 1.07 Q_{full}, where Q_{full} is the full-flow discharge calculated using the Manning equation (Brater et al., 1996).

Unsubmerged Entrance. When the culvert entrance is unsubmerged, the flow is either Type 4, 5, or 6. The flow type and the associated flowrate are determined by the following procedure:

1. Assume that the flow is Type 4, use Equation 4.184 to calculate the discharge, Q, and use Q to calculate the normal depth, y_n, and critical depth, y_c. If $y_n > y_c$ and the tailwater depth is less than y_c, then Type 4 flow is verified and the calculated discharge is correct.
2. Assume that the flow is Type 5, use Equation 4.186 to calculate the discharge, Q, and use Q to calculate the normal depth, y_n, and critical depth, y_c. If $y_n < y_c$ and the tailwater depth is less than y_c, then Type 5 flow is verified and the calculated discharge is correct.
3. Assume that the flow is Type 6, use Equation 4.188 to calculate the discharge, Q, and use Q to calculate the normal depth, y_n, and critical depth, y_c. If $y_n > y_c$ and the tailwater depth is greater than y_c, then Type 6 flow is verified and the calculated discharge is correct.

Design Considerations. The geometry of a culvert entrance is an important aspect of culvert design since the culvert entrance exerts a significant influence on the hydraulic characteristics of a culvert. The four standard inlet types are: (1) flush setting in a vertical headwall, (2) wingwall entrance, (3) projecting entrance, and (4) mitered entrance set flush with a sloping embankment. Structural stability, aesthetics, and erosion control are among the factors that influence the selection of the inlet configuration. The entrance loss coefficient, k_e, used to describe the entrance losses in most discharge formulae depend on the pipe material, shape, and entrance type, and can be estimated using the guidelines in Table 4.10.

Local drainage regulations often state the minimum culvert size (usually in the 30 to 60 cm range), with debris potential being an important consideration in determining the minimum acceptable size of the culvert. Some localities require that the engineer assume 25% debris blockage in computing the required size of the culvert. Both minimum and maximum velocities must be considered in designing a culvert. A minimum velocity in a culvert of 0.6 to 0.9 m/s at the design flow is required to assure self-cleansing. The maximum allowable velocity for corrugated metal pipe is 3 m/s (10 ft/s), and there is no specified maximum allowable velocity for reinforced concrete pipe (Debo and Reese, 1995), although velocities greater than 4–5 m/s are rarely used because of potential problems with scour. Outlet protection should be provided where discharge velocities will cause erosion problems. The most common culvert materials are concrete (reinforced and nonreinforced), corrugated aluminum, and corrugated steel. The selection of a

Table 4.10
Culvert Entrance Loss Coefficients

Culvert type and entrance conditions	k_e
Pipe, concrete:	
Projecting from fill, socket end (groove end)	0.2
Projecting from fill, square-cut end	0.5
Headwall or headwall and wingwalls	
Socket end of pipe (groove end)	0.2
Square edge	0.5
Rounded (radius = $1/12\,D$)	0.2
Mitered to conform to fill slope	0.7
End section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side- or slope-tapered inlet	0.2
Pipe, or pipe arched, corrugated metal:	
Projecting from fill (no headwall)	0.9
Headwall or headwall and wingwalls square edge	0.5
Mitered to conform to fill slope, paved or unpaved slope	0.7
End section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side- or slope-tapered inlet	0.2
Box, reinforced concrete:	
Headwall parallel to embankment (no wingwalls)	
Square edged on 3 edges	0.5
Rounded on 3 edges	0.2
Wingwalls at 30° to 75° to barrel	
Square edged at crown	0.4
Crown edge rounded	0.2
Wingwalls at 10° to 25° to barrel	
Square edged at crown	0.5
Wingwalls parallel	
Square edged at crown	0.7
Side or slope-tapered inlet	0.2

Source: USFHWA (1984a).

Table 4.11 Manning n in Culverts

Type of conduit	Wall and joint description	n
Concrete pipe	good joints, smooth walls	0.011–0.013
	good joints, rough walls	0.014–0.016
	poor joints, rough walls	0.016–0.017
	badly spalled	0.015–0.020
Concrete box	good joints, smooth finished walls	0.014–0.018
	poor joints, rough, unfinished walls	0.014–0.018
Corrugated metal pipes and boxes, annular corrugations	$2\frac{2}{3} \times \frac{1}{2}$ inch corrugations	0.022–0.027
	6×1 inch corrugations	0.022–0.025
	5×1 inch corrugations	0.025–0.026
	3×1 inch corrugations	0.027–0.028
	6×2 inch structural plate	0.033–0.035
	$9 \times 2\frac{1}{2}$ inch structural plate	0.033–0.037

Source: USFHWA (1984a).

culvert material depends on the required structural strength, hydraulic roughness, durability, and corrosion and abrasion resistance. In general, corrugated culverts have significantly higher frictional resistance than concrete culverts, and most cities require the use of concrete pipe for culverts placed in critical areas or within the public right-of-way. Recommended Manning n values for culvert design are given in Table 4.11.

Example 4.16

What is the capacity of a 1.22 m by 1.22 m concrete box culvert ($n = 0.013$) with a rounded entrance ($k_e = 0.05$, $C_d = 0.95$) if the culvert slope is 0.5%, the length is 36.6 m, and the headwater level is 1.83 m above the culvert invert? Consider the following cases: (a) free-outlet conditions, and (b) tailwater elevation 0.304 m above the top of the box at the outlet. What must the headwater elevation be in case (b) for the culvert to pass the flow that exists in case (a)?

Solution

Since the headwater depth exceeds 1.2 times the height of the culvert opening, then the culvert entrance is *submerged*. An elevation view of the culvert is shown in Figure 4.29.

(a) For free outlet conditions, two types of flow are possible: the normal depth of flow is greater than the culvert depth (Type 2), or the normal depth of flow is less than the culvert depth (Type 3). To determine the flow type, assume a certain type of flow, calculate the discharge and depth of flow, and see if the assumption is confirmed. If

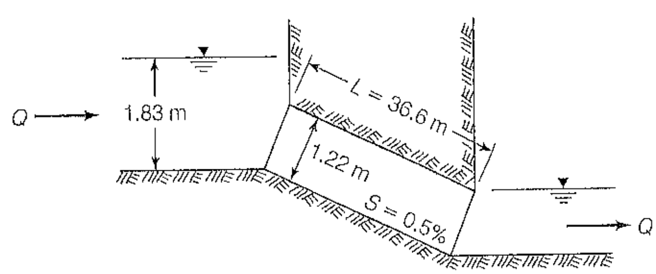


Figure 4.29 Elevation View of Culvert

the assumption is not confirmed, then the initial assumption is incorrect. Assuming Type 2 flow, then the flowrate equation, Equation 4.181, is given by

$$Q = A\sqrt{\frac{2g\Delta h}{19.62n^2L/R^{\frac{4}{3}} + k_e + 1}} \tag{4.189}$$

where Δh is the difference in water levels between the entrance and exit of the culvert [$= 0.183 + 0.005(36.6) - 1.22 = 0.793$ m], n is the roughness coefficient ($= 0.013$), L is the length of the culvert ($= 36.6$ m), and R is the hydraulic radius given by

$$R = \frac{A}{P}$$

where A is the cross-sectional area of the culvert, and P is the wetted perimeter of the culvert where

$$A = (1.22)(1.22) = 1.49 \text{ m}^2$$

$$P = 4(1.22) = 4.88 \text{ m}$$

and therefore

$$R = \frac{1.49}{4.88} = 0.305 \text{ m}$$

Substituting known values of the culvert parameters into Equation 4.189 gives

$$Q = (1.49)\sqrt{\frac{2(9.81)(0.793)}{19.62(0.013)^2(36.6)/(0.305)^{\frac{4}{3}} + 0.05 + 1}}$$

which simplifies to

$$Q = 4.59 \text{ m}^3\text{/s}$$

The next step is to calculate the normal depth at a discharge of 4.59 m^3/s using the Manning equation, where

$$Q = \frac{1}{n}AR^{\frac{2}{3}}S_o^{\frac{1}{2}}$$

and S_o is the slope of the culvert ($= 0.005$). If the normal depth of flow is y_n, then the area, A, wetted perimeter, P, and hydraulic radius, R, are given by

$$A = by_n = 1.22y_n$$

$$P = b + 2y_n = 1.22 + 2y_n$$

$$R = \frac{A}{P} = \frac{1.22y_n}{1.22 + 2y_n}$$

The Manning equation gives

$$4.59 = \frac{1}{0.013}\frac{(1.22y_n)^{\frac{5}{3}}}{(1.22 + 2y_n)^{\frac{2}{3}}}(0.005)^{\frac{1}{2}}$$

which simplifies to

$$\frac{(1.22y_n)^{\frac{5}{3}}}{(1.22+2y_n)^{\frac{2}{3}}} = 0.844$$

and solving iteratively for y_n yields

$$y_n = 1.25 \text{ m}$$

Therefore, the initial assumption that the normal depth is greater than the height of the culvert (= 1.22 m) is verified, Type 2 flow is confirmed, and the flow through the culvert is equal to 4.59 m^3/s.

(b) In this case, the tailwater is 0.304 m above the culvert outlet, and therefore the difference in water levels between the inlet and outlet, Δh, decreases by 0.304 m to $0.793 - 0.304 = 0.489$ m. The flow equation in this case, Type 1 flow, is the same as Equation 4.189, with $\Delta h = 0.489$ m, which gives

$$Q = (1.49)\sqrt{\frac{2(9.81)(0.489)}{19.62(0.013)^2(36.6)/(0.305)^{\frac{4}{3}} + 0.05 + 1}}$$

which simplifies to

$$Q = 3.60 \text{ m}^3/\text{s}$$

Therefore, when the tailwater depth rises to 0.305 m above the culvert exit, the discharge decreases from 4.59 m^3/s to 3.60 m^3/s.

When the headwater is at a height x above the culvert inlet and the tailwater is 0.305 m above the outlet, then the flow through the culvert is 4.59 m^3/s. The difference between the headwater and tailwater elevations, Δh, is given by

$$\Delta h = [x + 1.22 + 0.005(36.6)] - [1.22 + 0.305] = x - 0.122$$

The flow equation for Type 1 flow (Equation 4.189) requires that

$$4.59 = (1.49)\sqrt{\frac{2(9.81)(x - 0.122)}{19.62(0.013)^2(36.6)/(0.305)^{\frac{4}{3}} + 0.05 + 1}}$$

which leads to

$$x = 0.915 \text{ m}$$

Therefore, the headwater depth at the entrance of the culvert for a flow of 4.59 m^3/s is $1.22 + 0.915 = 2.14$ m. ■

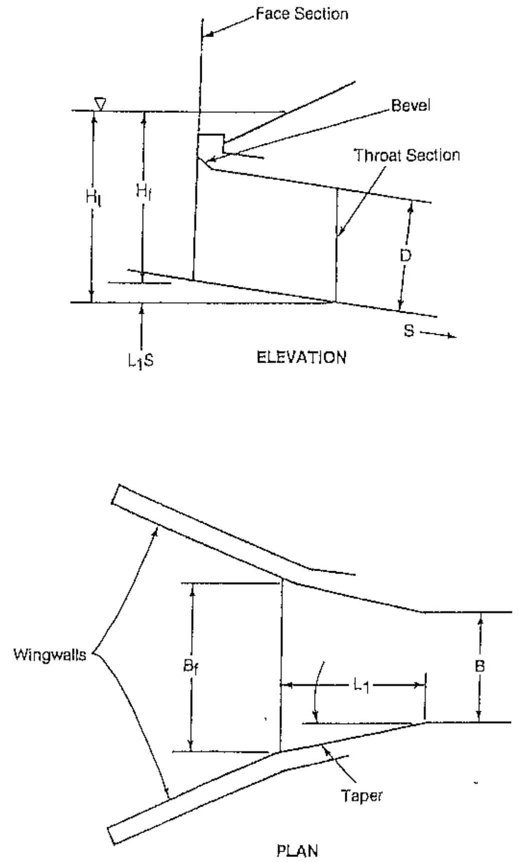


FIGURE 15.24 Typical side-tapered inlet detail. (From State of Florida Department of Transportation, 1987)

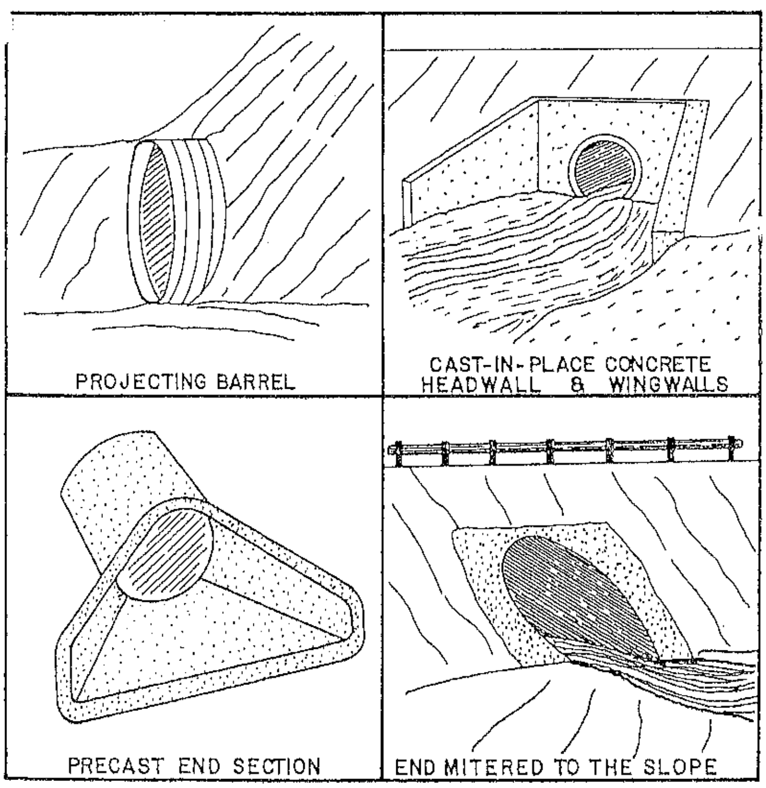


FIGURE 15.22 Four standard inlet types. (From Norman et al., 1985)

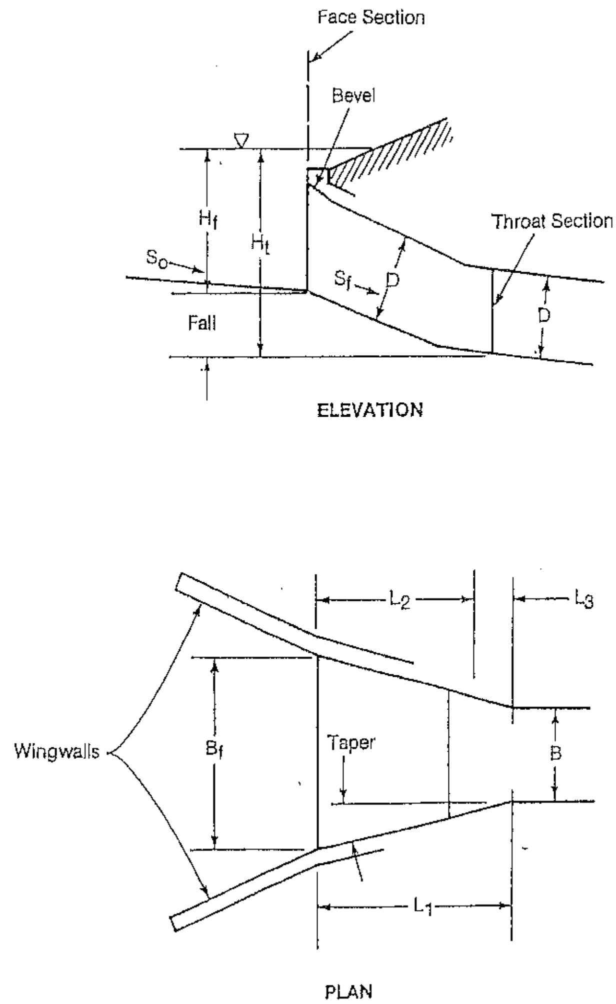
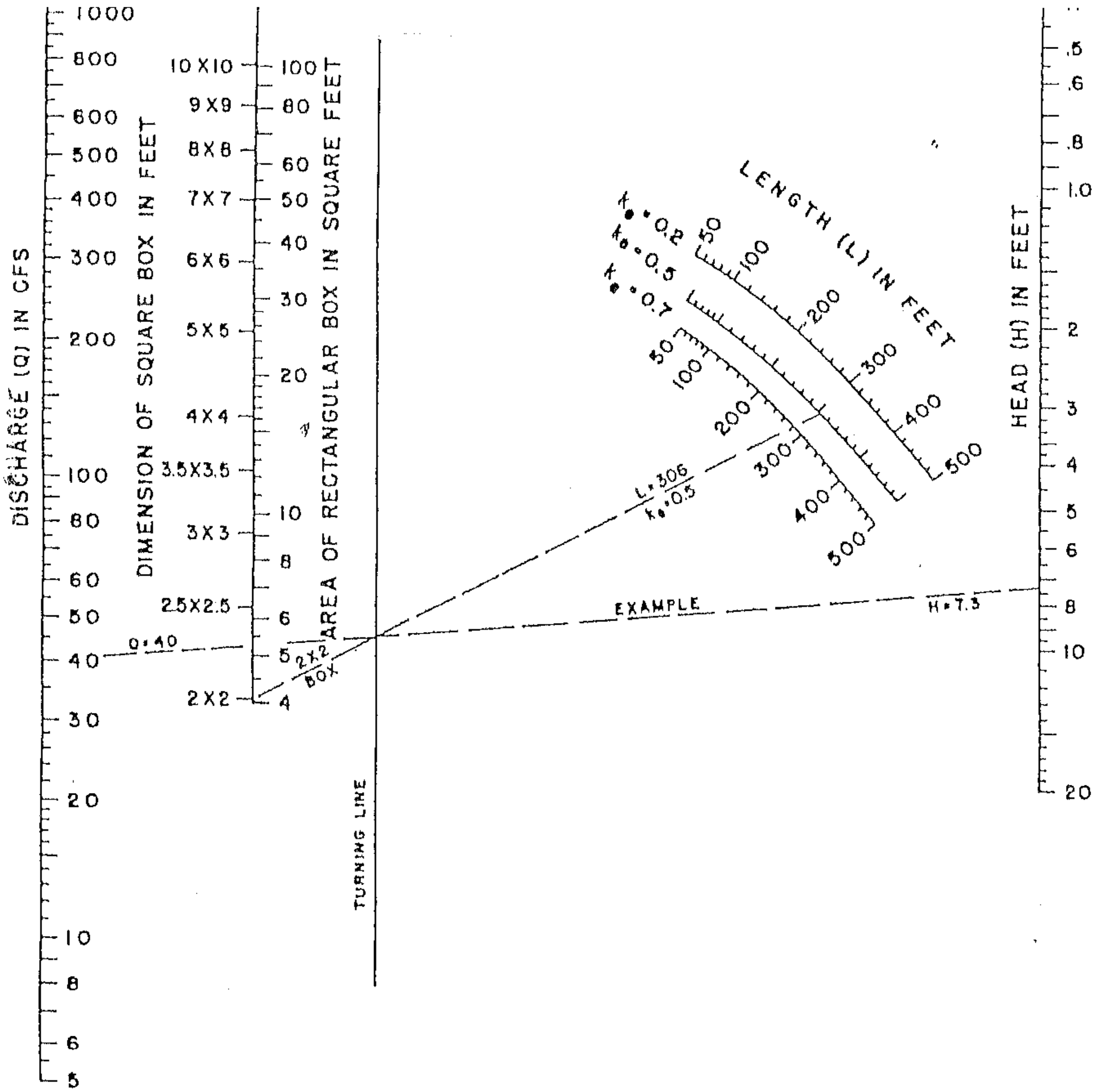


FIGURE 15.25 Typical slope-tapered inlet detail. (From State of Florida Department of Transportation, 1987)



Head for concrete box culverts flowing full ($n = 0.012$)